Volume I
Advanced and Modern General Relativity

Draft version

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Terminology and Notation

Here is a list of symbols.

$[\,,\,]$	commutator
$\{\,,\,\}$	Poisson bracket
$\dagger$	Hermitian conjugation
$:=$	definition
$\equiv$	identity
$\stackrel{*}{=}$	only true in a special coordinate system
iff	If and only if
η_{ab}	Minkowski metric
$\eta(x)$	test function of a variation of action
$\mathcal{A}$	space of gauge fields or area
$A_\mu(x)$	Yang-Mills connection
D_μ	covariant derivative
$\mathcal{M}$	spacetime manifold
$\mathbf{M}$	The Master consraint
$\hat{\mathbf{M}}$	The Master consraint operator
$\omega^{\alpha}_{\mu\beta}$	spin connection
$\mathcal{C}$	constraint surface in phase space
S	labells spin-network
s	equivalent class of spin-networks under the action of Diff denoted $s-$ knots
$s(S)$	denotes equivalent class S to which belongs
g_{ab}	spacetime metric
K_{ab}	extrinsic curvature of Σ
G_{ab}	Einstein tensor
T_{ab}	The energy-momentum tensor
e^a_I, E^a_i	tetrad and triad
$\mathcal{L}_t$	Lie derivative with respect to t
n_a	unit normal to Σ_t
N, $(\tilde{N})$	lapse function (density)
N^a	shift vector on Σ
$\Omega_{\alpha\beta}$	sympletic form
$\mathcal{A}/\mathcal{G}$	space of gauge fields moduli gauge transformations
$[A]$	gauge equivalence classe of the connection A
$\mathcal{HA}$	the holonomy algebra
$\overline{\mathcal{HA}}$	the completion of the holonomy algebra in the norm $\|f\| := \sup_{[A]\in\mathcal{A}/\mathcal{G}} \mid f([A]) \mid$
$\overline{\mathcal{A}/\mathcal{G}}$	spectrum of $\overline{\mathcal{HA}}$

Preface

Acknowledgments

Discussions with Tong,Pun Wai on parts of the proof of the singularity theorems.

Introduction

the beginning of the revolutionary contributions to physics by Einstein,

Paths through the report

Introductory book on general relativity [?]