Chapter 3

Semiclassical Analysis

3.1 Twisted Geometries

A basis of states for LQG is given by the spin network states. A spin network state has support on a graph Γ and determines a 3d "quantum geometry"; they are eigenvectors of geometric operators which determine the intrinsic geometry. The extrinsic gemetry (analogous to the conjugate momentum) is completely "spread out", due to the Heisenberg uncertainty principle.

To make contact with a semiclassical description of space, we can consider coherent states peaked (but not sharp) on both the intrinsic and exstrinsic geometry. A twisted geometry is a specific choice of "interpolating geometry", chosen among distcontinuous metrics.

Twisted geometries parametrise the phase space of loop quantum gravity on a fixed graph in terms of quantities describing the intrinsic and extrinsic geometry of a three dimensional triangulation dual to the graph.

To any graph and any holonomy-flux configuration, we can associate a twisted geometry: a discrete discontinuous geometry on a cellular decomposition space into polyhedra. Thanks to this result, the phase space of LQG on a graph can be visualised not only in terms of holonomies and fluxes, but also in terms of a simple geometric picture of adjacent flat polyhedra.

3.1.1 Hopf Map

Consider the space $\mathbb{C}^2$ of pairs of complex numbers (z_0, z_1). Define a bundle space by the equation

$$|z_0|^2 + |z_1|^2 = 1$$

This stands for the equation of a 3-sphere. Now consider the family of complex planes through the origin. Each such plane is given by qn equation of the form

$$Az_0 + Bz_1 = 0$$

where A and B are complex numbers (not both zero). Being 2-dimensional planes it intersects S^3 in a circle S^1. The different planes only meet at the origin, so no distsinct S^1's have no point in common.

We get the same plane $Az_0 + Bz_1 = 0$ if we muliply both A and B by the same complex non-zero number, so it is the ratio A/B that determines the different planes. The space of such ratios is the Riemann sphere.