Chapter 5

Physical Applications of LQG: Black Hole Entropy and Loop Quantum Cosmology

• Thermodynamics of Black holes and Hawking radiation.

• Microscopic sourse of black hole entropy. Entropy of black holes.

• Loop Quantum Cosmology.

• Inflation From Loop Quantum Cosmology.

• Consistent histories interpretaion.

• Relational quantum cosmology.

5.1 Introduction

5.2 Thermodynamics of Black holes and Hawking radiation

Second law of black hole mechanics

$$\delta\mathcal{A} \geq 0 \tag{5.1}$$

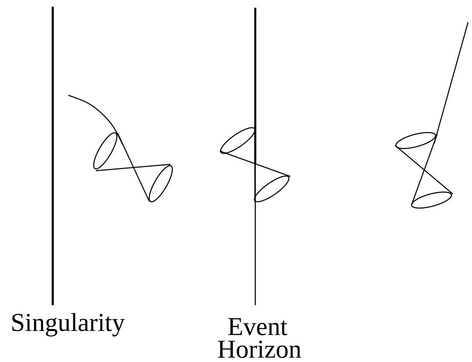

Figure 5.1: .

Second law of thermodynamics

$$\delta S \geq 0 \tag{5.2}$$

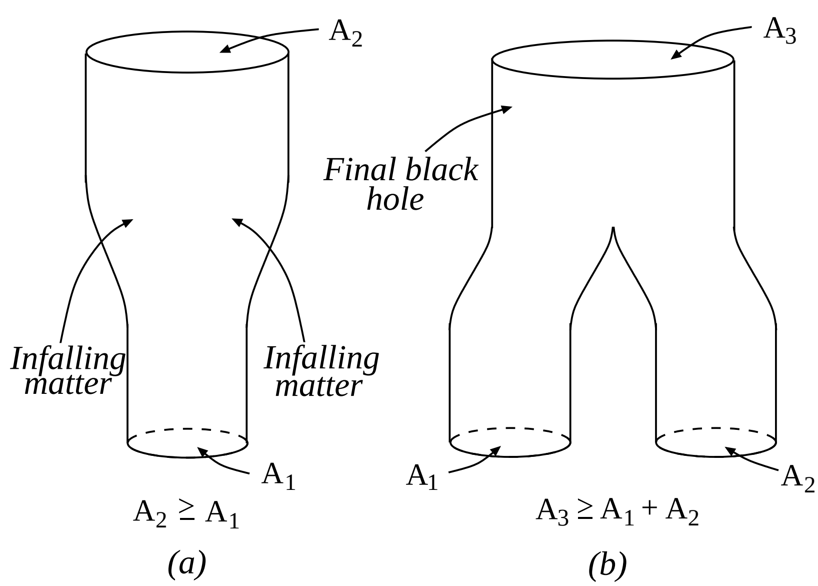

Figure 5.2: areaincrease.

First law of black hole mechanics:

$$\delta E = \frac{\kappa}{8\pi}\delta\mathcal{A} + \Omega\delta J + \Omega\delta\mathcal{Q} \tag{5.3}$$

It was immediately noticed by Bardeen, Carter and Hawking (1973) that there was a close similarity between these laws of black hole mechanics and the usual laws of thermodynamics, with κ proportional to the temperature and A proportional to the entropy.

First law of thermodynamics:

$$\delta E = T\delta + P\delta V \tag{5.4}$$

Zeroth law of black hole mechanics

κ is the same everywhere on the horizon of a time-independent black hole.

Zeroth law of thermodynamics

T is the same everywhere for a system in thermal equilibrium.

When the expansion goes negative, that is when the geodesics are converging the attractive nature of gravitation means that they will intersect within a finite affine parameter time.

However, it was originally thought that this could only be an analogy, since if a black hole really had a nonzero temperature, it would have to radiate and everyone knew that nothing could escape from a black hole. This view changed completely when Hawking (1975) showed that if matter is treated quantum mechanically, black holes do radiate. This showed that black holes are indeed thermodynamic objects with a temperature and entropy given by

$$T_{Bh} = \frac{\hbar\kappa}{2\pi}, \qquad S_{Bh} = \frac{A}{4\hbar} \tag{5.5}$$

5.3 Hawking Radiation

See [130] for note containing a step-by-step presentation of Hawking's calculation.

Rough idea:

See [130] for note containing a step-by-step presentation of Hawking's calculationd. In 1974, Steven Hawking showed that black holes do radiate quantum mechanically, thereby shinking in area. Only matter was treated quantum mechanically; there were no quanta of geometry.

$$\left(i\frac{\partial}{\partial x^\mu} - eA_\mu\right)^2 \psi(x) = m^2\psi(x). \tag{5.6}$$

$$i\frac{\partial\varphi}{\partial\lambda} = -\frac{1}{2}\left(i\frac{\partial}{\partial x^\mu} - eA_\mu\right)^2 \varphi \tag{5.7}$$

Because the functions A_μ are independent of λ, (5.7) has solutions of the form $\varphi(x,\lambda) = \exp(im^2/2)\psi(x)$ with satisfying (5.6).

$$\mathcal{L} = \frac{1}{2}\left(\frac{dx^\mu}{d\lambda}\right)^2 + e\frac{dx^\mu}{d\lambda}A_\mu \tag{5.8}$$

$$G(x_A, x_B) = \int_0^\infty e^{-im^2\lambda/2} G(x_A, x_B; \lambda)\, d\lambda \tag{5.9}$$

with $G(x_A, x_B; \lambda)$ given by the path integral in the Schwarzschild metric

$$G(x_A, x_B; \lambda) = \int_{x(0)=x_A}^{x(\lambda)=x_B} \mathcal{D}[x] \exp\left(i\int_0^\lambda \frac{1}{2} g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda}\, d\lambda\right). \tag{5.10}$$

$$\mathcal{H}_s \otimes \mathcal{H}_B \tag{5.11}$$

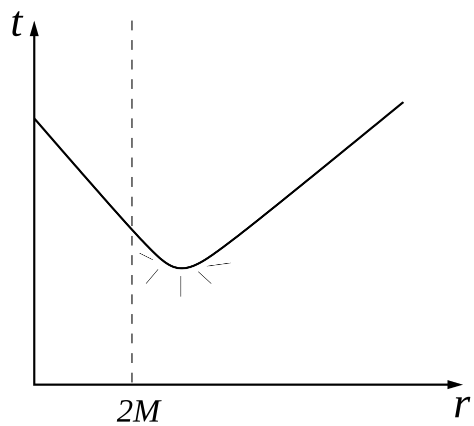

Figure 5.3: Reflects forward in time by the strong gravitaional field outside the event horizon.

Feynmann told us that we can interpret particles that going backwards in time as anti-particles (see details on page 1560). So anti-particles get swallowed by the black hole leaving a particle which has a chance of escaping off to infinity. That is, the black hole evapourates particles reducing its mass.

5.4 Laws of Thermodynamics from Statistical Mechanics

There is an area of theoretical physics that gives us indirect information on quantum gravity: black hole thermodynamics. The great power of thermodynamics to put con-

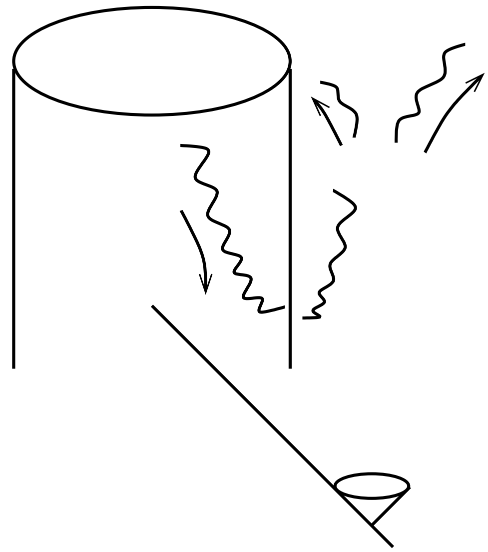

Figure 5.4: timereflection2. The antiparticle mode falling into the black hole can be interpreted as a particle travelling backwards in time, form the singularity down to the horizon.

straints on theoretical constructions, and even provide precise quantitative indications on microscopic theories is well known: quantum mechanics itself was born to a large extent in order to satisfy thermodynamical consistency requirements (Planck's spectrum, solid state...).

The first law of thermodynamics is an expression of conservation of energy including internal energy, heat and work.

We can just follow the usual argument that under some ergodicity hypothesis tells us what are the equilibrium states, if they exist.

In statistical mechanics, the relation $dS = dE/T$ is derived as a relation between nearby equilibrium states, not as a description of a dynamical process of approach to equilibrium.

$$\sum_i n_i = N; \tag{5.12}$$

$$\sum_j m_j = M. \tag{5.13}$$

and

$$\sum_i n_i \epsilon_i + \sum_j m_j \eta_j = E. \tag{5.14}$$

We write down the number of stationary states of the two ensembles combined; this number is given by

$$P = \frac{N!}{n_1!n_2!\dots n_i!} \times \frac{M!}{m_1!m_2!\dots m_j!} \tag{5.15}$$

This is just the product of the number of stationary states in ensemble A with the number of stationary states of ensemble B. We are treating the two systems as if any stationary state of the composite system is produced by the combination of any stationary state of one system with any stationary state of the other system, that is, as if they were isolated (apart from the condition that the total energy is conserved).

We want the values of the n_i's and of the m_j's which make P a maximum subject to the constraints.

$$\delta \ln P = -\delta \ln \sum_i n_i! - \delta \ln \sum_j m_j! = 0 \tag{5.16}$$

subject to the condition that

5.4.1 First Law

What do we mean by deriving the first law from a microscopic description? In a typical setting of a system in thermodynamic equilibrium, the statistical description of the system is given by a density matrix ρ. Let's fix on the canonical ensemble, so $\rho = \exp(-\beta\mathcal{H})/Z$, where β is the inverse temperature, $\mathcal{H}$ is the Hamiltonian, and Z is the trace of $\exp(-\beta\mathcal{H})$. Suppose some energy is added to the system, but in such a way that $\delta\rho \ll \rho$. Then varying the entropy $S = -Tr(\rho \ln \rho)$ and the mean energy $< E >= Tr(\rho\mathcal{H})$, one finds the relation

$$\delta S = \beta\delta < E >, \tag{5.17}$$

hence the thermodynamic relation $dE = TdS$ the first law. Thus once we have a system whose density matrix is a canonical ensemble, we have a derivation of the first law.

Let the number of system plus exterior states characterized by E_s and E_e be N. By ergodicity,

$$N = N_e(E_e) \times N_s(E_s),$$

where $N_e(E_e)$ is the number of states of the exterior with Energy E_e and similarly for $N_s(E_s)$. Consider a different macroscopic equilibrium state obtained by an adiabatic process that decreases E_e by dE and increases E_s by a corresponding amount. The corresponding number of microstates will be

$$N(dE) = N_e(E_e - dE) \times N_s(E_s + dE).$$

If our original state is the most probable, N must be the d maximum of $N(dE)$, obtained for $dE = 0$. Hence

$$\frac{d}{dE} \ln N(E) = 0.$$

Namely

$$\frac{dS_e(E_e)}{dE_e} = \frac{dS_s(E_s)}{dE_s} \equiv \frac{1}{T}.$$

That is, an equilibrium situation is characterized by a macroscopic parameter T which must be the same for the exterior and the surface and such that $dE = TdS$.

5.5 Black Hole Statistical Mechanics

Now, black hole thermodynamics derives a surprising set of simple laws just from classical general relativity and quantum field theory in curved spacetime. These laws have not been experimentally tested, but are very well motivated. However, they are thermodynamical "phenomenological" laws, and their derivation from first principles requires a quantum theory of gravity.

Phase space of a system is a space each of whose points represents an entire physical state. A single phase-space point provides all the microscopic coordinates of all individual constitutes of the physical system. We group together the all the states which look alike from the macroscopic properties.

Thus, for two completely independent physical systems, the total entropy of the two systems combined will be the sum of the entropies of each system separately.

The statistical ensemble is the region of phase space over which the system could wander if it were isolated, namely if it did not exchange energy with its surroundings.

from theomodynamics we see the temperature and entropy arises from underlying statistical mechanics. What microstaes are responsible for black hole thermodynamics?

a classical, stationary black hole is determined completely by its mass, charge, and angular momentum, with no room for additional microscopic states to account for thermal behaviour.

If black hole thermodynamics has a statistical mechanical origin then the relavent states must therefore be non-classical.

In the classical theory a realistic black hole with vanishing cahrge and angular momnetum evolves very rapidly towards the Schwarzschild solution, by rapidly radiating away all excess energy. In the qunatum theory, the Heisneberg principle prevents the hole from converging exactly to a Schwarzschild metric.

A microstate is not given by the Schwarzschild metric, but by some complicated time-dependent non-symmetric metric.

such time-dependent non-symmetric microstates of the geometry into account is essential for a statistical understanding of the thermal behavior of black holes

The interior degrees of freedom of the black hole are indistguishable to an exterior observer - classically because there is a causal barrier at horizon stops the interior effecting the exterior[1], hence these degrees of freedon do not contribute to the entropy and so don't effect the energy exchange between the black hole and the exterior.

5.5.1 First Law

There is no energy conservation that allows us to follow the usual statistical mechanical line of thinking. In fact, a strict attachment to the nonrelativistic notion of energy is not useful in the general relativistic context, where energy is a more delicate notion. Now, what is the role of energy conservation in the microcanonical setting? It is to reduce the region of phase space where the system is free to wander (ergodicity then tells us the systems are all over the allowed region). Now, in general relativity there is no analogous energy conservation. Thus, we have to look for a dynamical input from the Einstein equation that can play the same role that energy conservation plays in the nonrelativistic context. We have precisely what we need: the theorem stating that

> "the area of the horizon does not decrease".

The limit in which we disregard quantum effects, the area is compatible with the Einstein dynamics, as a parameter characterizing an equilibrium state. This theorem captures the relevant information from the general relativistic dynamics needed for understanding the statistical mechanics of the horizon. They play the same role as the microscopic energy conservation in the usual nonrelativistic thermodynamic context.

[1]assuming that the horizon is a strict causal barrier even at the quantum level.

The area is the only quantity that (classically) cannot decrease, and therefore the ensemble is formed by all the microstates with a certain area. The Einstein equation tells us also that if we throw a certain amount of energy dE across the horizon, then the area will increase by a certain amount. Hence a change in area is connected to a change in external energy. If we throw an energy dE inside the hole, then its horizon will fluctuate over a different ensemble, characterized by a larger area. There is no need to think that there is an actual "Energy" that remains on the surface degrees of freedom without continuing inside the hole. The Einstein dynamics allows us to apply the usual statistical mechanics logic, with just what is needed: a condition on the set of microstates in which the system can be. This is an answer to the question about the fact that energy enters the hole and therefore cannot thermalize on the surface degrees of freedom.

5.6 Entropy of Black Holes (Isolated Horizons)

Early idas: Smolin (95) Krasnov (96), Rovelli (96) More refined treatment: Ashtekar, Baez, Krasnov, Corichi, Lewandowski, Beetle. Fairhurst, Dreyer, Krishnan(99-00).

(Easy to read presentation: Ashtekar gr-qc/9910101)

They used the full theory but probes consequences of quatum geometry which are not sensitive to the full quantum dynamics.

Standard treatments isolated black holes are represented by stationary solutions of the field equations. This simple idealization it seems over restrictive.

Isolated horizons are generalizations of the event horizon of stationary black holes to physically more realistic situations. The generaliztion is in two directons. First, while one needs the entire space-time history to locate an event horizon, isolated horizons are defined using properties of space-time at the horizon. Second, although the horizon itself is stationary, the outside space-time can contain non-stationary fields and admit radiaton.

the event horizons can only be identified after knowing the complete evolution of the space-time. Consider fig (a) marginally trapped surfaces and would form part of the event horizon as nothing else hapens. Suppose instead (fig(b)) that after a long time, that a thin shell of mass δM collapses. Then Δ_1 would no longer be part of the event horizon which would lie slightly outside Δ_1.

Physically, it should be sufficient to impose boundary conditions at the horizon the horizon that ensure only the back hole itself is isolated. That a spacetime possess such a boundary defines a certain subset of the phase space of full general relativity.

Yet, the structure available on isolated horizons is sufficiently rich to allow a natural extension of the standard laws of black hole mechanics. Finally, cosmological horizons to which thermodynamical considerations also apply are special cases of isolated horizons. One can The precise notion of an isolated horizon can be arrived at by extracting

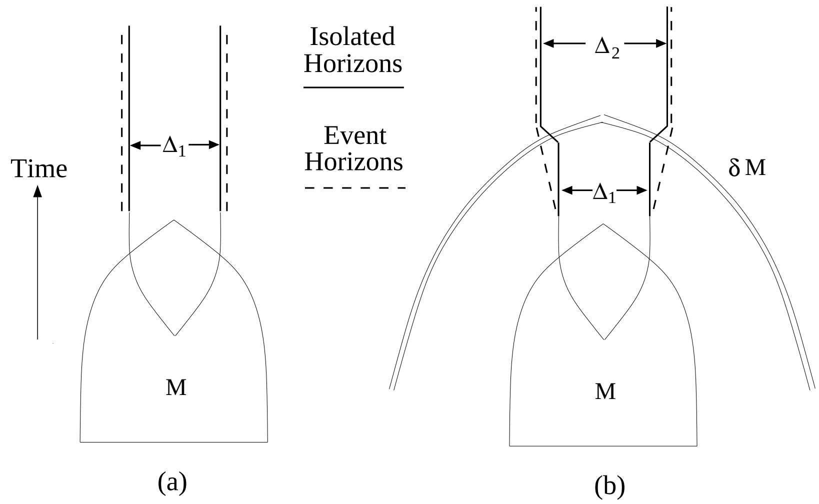

Figure 5.5: a) A spherical star of mass M undergoes collapse. b) Later, a spherical shell of mass δM falls into the resulting black hole. With Δ_1 and Δ_2 are both isolated horizons, only Δ_2 is part of the event horizon.

from the definition of a Killing horizon the minimum conditions necessary for black hole thermodynamics.

The first task consisits on choosing a surface that is reasonably close in its definition to a horizon, but that is an actual observable of the theory. This was accomplished by Ashtekar and collaborators through the notion of "isolated horizon".

Since spherical symmetry is assumed only at the horizon, the concept embodies the natural idea that even in the dynamical spacetimes with radiation, even arbitrarly close to the horizon.

Δ is a null 3-surface $R \times S^2$. With zero shear and expansion.

Field equations hold on Δ.

(1) H is null and $\approx R \times S^2$

(2) S is outer marginally trapped

(3) no gravitational radiation or matter falls in H

(4) the area A of S is time-independent.

If one studies in detail the Einstein action with the set of boundary conditions above, one finds that for the action to be differentiable one needs to add boundary terms. The boundary terms have the form of the integral of a Chern-Simons form built with the Ashtekar connection.

One can then construct a quantum theory with Hilbert space,

$$H = H_{bulk} \otimes H_{surface} \tag{5.18}$$

These two spaces are not entirely disconnected, it turns out that the "level" k of the Chern-simons theory is determined by the bulk.

One then wishes to consider a microcanonical ensemble, in terms of the area, $(a_0 - \delta a, a_0 + \delta a)$. The quantum boundary conditions dictate that for a given state in the bulk that punctures P times the surface, the Chern-Simons state has its curvature concentrated at the punctures, one therefore has,

$$H_{surface}^{phys} = \oplus_P H_{surface}^P \tag{5.19}$$

Each puncture adds an element of area $8\pi\beta$ and introduces a deficit angle of value $2\pi m_i/k$ Where m is in the interval of $[-j_i, +j_i]$ and k is the "level" of the Chern-simons theory.

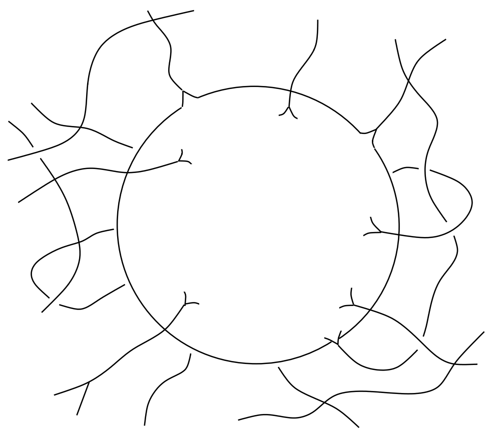

Figure 5.6: Quantum Horizon. Polymer exicitations in the bulk puncture the horizon, endowing it with quantized area. Intrinsically, the horizon is flat except at punctures where it aquires a quantized deficit angle. These angles add up to 4π.

The picture of the quantum geometry of the horizon that appears is that it is flat except at punctures where the lines of gravitational flux "pull" the surface up and introduce curvature. Let us state more precisely what we mean by this quantity: this curvature is defined in an intricsic way - imagine starting at a point p and moving a geodesic distance ϵ in all directions. What you would form is the closests thing to a "circle" on this surface. If a space is flat, the circumference C of the circle is $C = 2\pi\epsilon$. However, on a curved surface the circumference would be slightly smaller or greater that this depending on whether the surface has positive or negative curvature at the point p. Gauss suggested the following definition for curvature

$$K_{Gauss} = \lim_{\epsilon} \frac{6}{\epsilon^2}\left(1 - \frac{C}{2\pi\epsilon}\right). \tag{5.20}$$

The curvature of the isolated horizon is rather sigular: consider the diagram fig (5.7). The circle is flat. We glue together the edges to form a cone. The surface is flat yet by (5.20) there is curvature - the curvature must all be concentrated at the tip of the cone, whats known as a *canonical singularity* (vortices in a two-dinensional fluid with angular momentum however the vorticity is zero everywhere except at the centre).

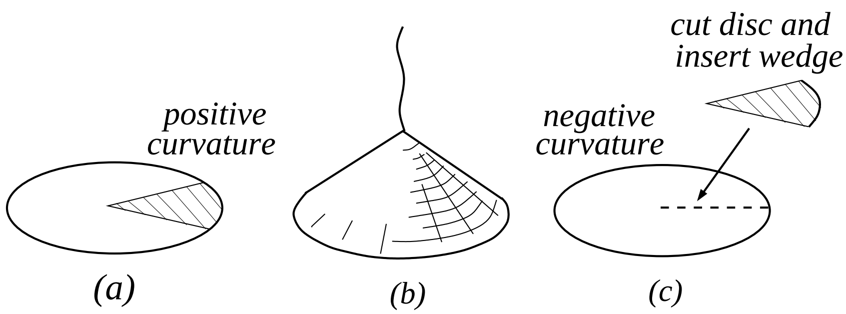

Figure 5.7: Deficit angle in horizon 2-geometry. (a) surface is flat (b) A 3-dimensional perspective: a bulk polymer excitation "exerts a tug" on the horizon exciting a quantum of curvature. (c) A cut is made in the disc and a wedge is incerted corresponds to negative curvature by (5.20).

At each puncture there is a quantized deficit angle. These deficit angles add up to endow the horizon with a 2-sphere topolgy. For a solar mass black hole, a typical horizon state would have 10^{77} punctures, each contributing a tiny deficit angle. So, although the quantum geometry is distributional, it can be well approximated by a smooth metric. Suppose one has a black hole. The area of the horizon will have an eigenvalue S. There are many rearrangements of the spin networks that yield the same eigenvalue. Counting the number of these quantum states gives a measure of the entropy of the black hole.

In the quantum theory neither the intrinsic geometry nor the horizon geometry are frozen; neither is a classical field. Each is allowed to undergo quatum fluctuations but because of the operator equation relating them, they have to flutuate in tandem.

They do not have complete control over the quatum Hamiltonian constraint. To proceed they make the rather weak assumptionthat for each microstate of the horizon geometry there corresponds at least one spin network state in the bulk that solves all the constraints of quantum GR

For large black holes the microsatates wich assign to each puncture the smallest quantum of area ($j = 1/2$) dominate the counting as they maximise the number of punctures required for a given horizon area. In this case, each contribution to the area of the horizon is $4\pi\sqrt{3}\gamma$. For each puncture m takes two possible values $\pm 1/2$. For n punctures, we have $AS = 4\pi\sqrt{3}\gamma n$ and entropy

$$\text{Entropy} \;\; S \approx \ln(2^n) = \frac{\ln(2)}{4\pi\sqrt{3}\gamma} A \tag{5.21}$$

We have agreement with Hawkings formula $S = A/4$ if we take $\gamma = \frac{\ln 2}{\pi\sqrt{3}}$. The smallest quantum of area is then $4\ln(2)$.

We have made use of the assumption that for each set of punctures on the horizon that there is at least one solution to the hamiltonian constraint. This allowed us to write

$\mathrm{S} = \ln N + n\ln 2 \approx n\ln 2$

where N is the number of bulk solutions, with $N > 0$. Since thermodynamic entropy is defined only up to an additive constant, we may argue that the bulk states do not play any role in black hole thermodynamics.

It is good that the result (log) is proportional to the area (without the tight boundary conditions one gets results proportional to square root of area, for instance). The results depends on a free parameter, the Immirzi parameter γ.

Considering black holes with elecromagnetic or dilatonic charge one finds that one gets the correct result for the same value of the Immirzi parameter. The result has been extended to rotating black holes apparently! In this asnalysis, all black holes and cosmological horizons are treated in a universal fashion; there is no restriction , e.g. to near-extremal black holes.

To determine the Immirzi parameter γ we called on Hawkikng's result that seemed like cheating. However, it has been argued that by looking at the quasinormal damped modes of a classical black hole one is able to derive the quanta of area in a different way. They conclude that $\Delta A = 4\ln(3)$. But this would correspond not to a minimum value $j_min = 1/2$ but to $j_min = 1$. There is the sugestion that one should think of a conserved fermion number being assigned to each spin-1/2 edge so that

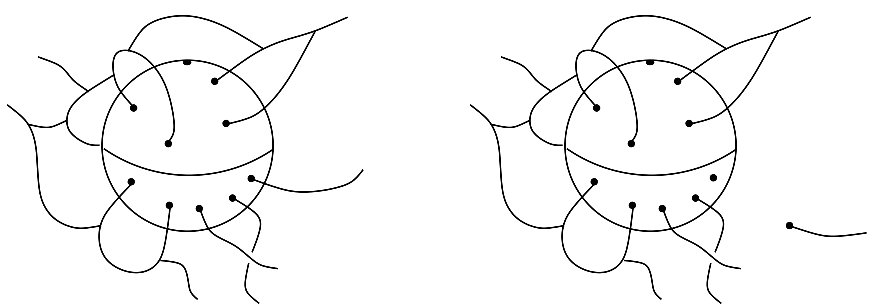

Figure 5.8: An example of the emission in which one of the flux lines breaks, with one of the ends falling into the black hole and the other escaping to infinity.

In 1974, Steven Hawking showed that black holes do radiate quantum mechanically, thereby shinking in area. This is a strong hint that the geometrical area of a black hole horizon can be converted into matter. Only matter was treated quantum mechanically; there were no quanta of area. With quantum geometry, they could re-examine the situation. Now, it is literally true that the black hole horizon aquires its area through its interactions with polymer geometry. In the Hawking process, quanta of area are converted to quanta of matter.

Details: Illustrative example: Maxwell Field in Minkowskian space-time outside $r = r_0$.

$$S = \int_{\mathcal{M}} d^4x \epsilon^{\mu\nu\sigma\rho} F_{\mu\nu} F_{\sigma\rho} = -\frac{1}{4} \int d^4x \; F_{\mu\nu}(x) F^{\mu\nu}(x) \tag{5.22}$$

Boundary conditions:

1. At ∞: Standard ones.
2. At Δ: $F^{(\Delta)}_{\mu\nu} = \lambda \, {}^* F^{(\Delta)}_{\mu\nu}$

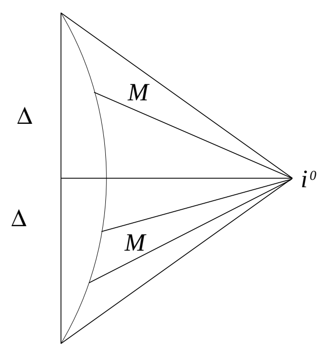

Figure 5.9: Maxwell Field in Minkowskian space-time outside $r = r_0$.

Then to make the action differentiable we have to add a surface term.

$$S = \int_{\mathcal{M}} d^4x \epsilon^{\mu\nu\sigma\rho} F_{\mu\nu} F_{\sigma\rho} + \underbrace{\frac{\lambda}{2} \oint_{\Delta} \epsilon^{\mu\nu\sigma} F_{\mu\nu} A_\sigma}_{U(1)\; Chern-Simons} , \tag{5.23}$$

or in differerntial geometry language

$$S = \int_{\mathcal{M}} F \wedge {}^*F + \frac{\lambda}{2} \underbrace{\oint_{\Delta} F \wedge A}_{U(1)\ \text{Chern}-\text{Simons}} \tag{5.24}$$

Chern-Simons for Maxwell field

5.6.1 A new Hint: Quasinormal Modes of Black Holes

[155]

Corrections to counting

In [138], [139] it is shown that the initial calculation of the blackhole entropy was actually incorrect: there was a miscounting of the microscopic states contributing to the entropy as states labeled with higher spins also contribute to the entropy calculation.

one has that [139]

$$\frac{\ln 2}{\pi} \leq \gamma_1 \leq \frac{\ln 3}{\pi}. \tag{5.25}$$

Further issues:

• The Barbero-Immirzi pararmeter remains somewhat puzzling. quasinormal modes of black holes, still ...

• It is largley kinematic and doesn't address dynamical issues to do with black hole formation or black hole evapouration

• It does not address "what happens at the singularity.

See zero mass light shells [].

These last two issues are also being studed using the techniques which have already been applied to the subject of the next section.

5.6.2 Entropy of Rotating and Axially-Symmetric Distorted Blackholes (Type II Isolated Horizons)

the geometry is assumed to be axisymmetric. These are more representative of equilibrium states of *generic* horizons in realistic astrophysical and comological situations.

5.7 Black Hole Information Paradox

$$-i\hbar\frac{\partial}{\partial t} = \mathcal{H} \tag{5.26}$$

conservation of probabilities requires evolution to be *unitrary*.

In GR there is no time evolution

$$0 = \mathcal{H} \tag{5.27}$$

Probabilities preserved with respect to what? there is no time evolution - obseverables don't depend on the coordinate time and so there is no reason why the Hamiltonian needs to be unitrary.

We do have relational evolution. Since there is no such thing as a idel clock - the variable representing the clock too is subject to quantum fluctuations. Hence, there is a *natural decoherence*. We must answer wether the decoherence comming from the evapouration of the black hole is greater than the natural decohernce.

The system has an unitary evolution in n. At t cannot br perfectly correlated with n, even in the semi-classical regime of the clock, the evolution in t is not perfectly unitary. In fact one can show that the density matrix evolves according to

$$\frac{\partial}{\partial t}\rho_2 = -i[\mathcal{H}_2, \rho_2(t)] - \sigma[\mathcal{H}_2, [\mathcal{H}_2, \rho_2(t)]]. \tag{5.28}$$

This equation was first proposed by Milburn based on phenomenological arguments, and is a particular tpye of non-unitary evolutions considered by Lindblad.

Their derivation allows to estimate σ that is of order of the Planck time.

$$\rho_{2_{nm}}(t) = \rho_{2_{nm}}(0)e^{-i\omega_{nm}t}e^{(-\sigma(\omega_{nm})^2)t} \tag{5.29}$$

This equation doesnot violate the conservation of energy like Hawking prosal for information loss. One could exect to confirm this type of equation by studying some mesoscopic quatum systems.

Information loss problem in Black Holes

It provides a new and very effective mechanism for treating the information loss problem in Black Holes.

They have shown that for any Black hole bigger than 600 Plank masses the information loss induced by our equation is enough to dissipate all the black hole information before to its evaporation.

For very small black holes, Hawking's semi-classical analysisis not valid.

5.7.1 Singularity Avoidance

Quantum Blackholes

Using ideas first developed in the context of quantum cosmology, effects of the quantum nature of geometry on blackhole singularities have been recently analyzed [156] (Ashtekar and Bojowald in preparation). It is found that the black hole singularity is resolved but the classical spacetime dissolves in the Plank regime.

Singularity avoidance by collapsing shells in quantum gravity

Quantum Theory of Gravitation Collaspe - Zero Mass Light Shells

To investigate singularity we have to find a different approximation to WKB.

Isolated Horizons and Dynamical Horizons and Their Application [137]

5.8 Black Holes in Full Quantum Gravity

We would like to study black holes in the full non-perturbative quantum theory, without recurring to semiclassical considerations. Here a proposal is given of a definition of a quantum black hole as the collection of the quantum degrees of freedom that do not influence observables at infinity. From this definition, it follows that for an observer at infinity a black hole is described

So far, however, the description of black holes has relied on some mixture of quantum theory and classical analysis of black hole geometry: for instance, one can characterize a black hole classically, and then quantize the part of the classical theory phase space that contains the black hole. Is it possible, instead, to describe black holes entirely within the non perturbative quantum theory of spacetime?

In this section a suggestion is given for a direction for answering this question. We propose a simple definition of a quantum black hole within the full quantum loop theory, as a region of a spin network which is not "visible" from infinity. This is in the same spirit of the global analysis that is possible in classical general relativity, where properties of horizons and black holes can be obtained by studying their implicit definition, even without being able to solve the equations of motion and writing the metric explicitly

From the point of view of the outside observer, a black hole behaves as a possibly (gigantic) single interwiner, which interwines all the links puncturing its horizon.

The notion of a horizon used here is based on the traditional one (the boundary of the past of future null infinity), and has the same limitations. It would be interesting to find an extension of the above construction that could capture also the notion of an isolated horizon. In particular we could extend the result presented here also to the scenario where information is recovered during, or at the end, of the Hawking evaporation, and where, according to the traditional definition, there is no horizon.

Another suggestion

To fully understand the final evolution of black holes one needs a description...

[145] do not allow for the analysis of black hole evaporation.

Formulate quantum versions of the horizon boundary conditions and impose them at the quantum level so that they capture the intuiative idea of that classical black holes are defined by the presence of surfaces from which light cannot escape outward. They can used to identify quantum staes that describe quantum black hole. They allow for a defintion of

various dynamical processes of quantum black holes such as formation, mergers and evaporation.

5.8.1 Quantum geometry and the Schwarzschild singularity

[162]

5.9 Strings and Black-Hole Entropy

Andrew Strominger and Cumrum Vafa

Extremal brane systems turn out to share many properties with extremal black holes. In particular, the thermodynamical properties of the two systems are identical.

The original calculation pertained to 5-dimensional spacetime. Later results do apply to 4-dimensional spacetime, but the initial overblown proclamations were elicited by the original 5-dimensional calculation.

All "these string theory string-theoretical results referred only to the extremely special limitimg case of an 'extremal hole' (or to pertubations away from this) for which the Hawking temperature is zero - and where the hole involves additional supersymmetric Yang-Mills-type fields which have no clear justification from known physics".

It has not been settled whether the results correspond to a relationship between the two systems that is just an accidental consequence of the fact that both have a lot of extra symmetry and do not lead to general insights about black holes, or whether on the contrary, that all black holes can be understood using the same ideas, and that the extra symmetries present in special cases simply allow us to calculate more precisely.

Moreover, the calculations were performed in flat space, where there is no actual event horizon. In the calculation, you begin by turning off the gravitational force (you do this by putting the string coupling constant to zero). It is conjectured that they would become black-holes if the gravitational force is slowly turned on. But the problem is that it has been very difficult to define consistent string theories (or any compellingly proof that such a string theory exists) where the graviational field changes in time.

Do the black hole results of string theory concern actual black holes or do they just concern ensembles of states in free string theory in flat spacetime, with gravitational coupling turned off?

5.10 Quantum Cosmology

We make no attempt to make sense of the quantum theory of a single universe in which the notion of an external observer has no place, and usual probabilistic interpretation of the wavefunction is questionable.

Take semi-classical solution an evolve backwards using the Wheeler-De Wit equation you still end up with a singularity [].

Restrict techniques of loop quantum gravity to cosmologies with compact (isotropic) and homogeneous spacelike silces. Take Thiemann's ideas on the Hamiltonian constraint and apply them to mini super space quantization.

5.10.1 Classical theory

$$d\tau^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2). \tag{5.30}$$

Where a is the scale factor, and in solving Einstein's equations the task is to determine a as a function of t.

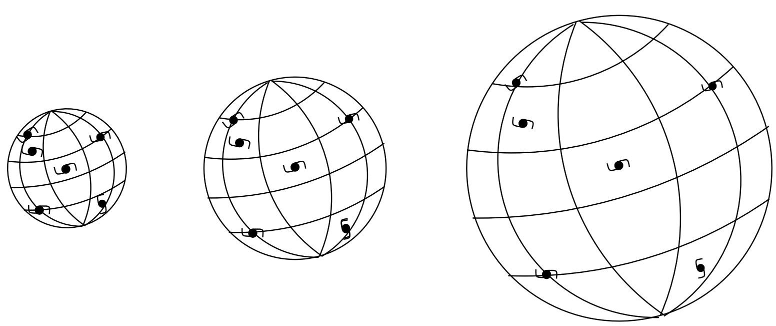

Figure 5.10: Expansion of closed universe.

The space itself is brought into being at the big bang. It is only for visualization purposes that the sphere sits in the 3-dimensional Euclidean space; we imagine we are a 2D creature living on the sphere and that there is no “ambient space” (it is not contained in a further space - here the Euclidean space), in which the sphere is embedded. a hypershere - a 3-dimensional version of a spherical surface, we have the ability to circumnavigate the univeerse by travelling always in the same direction until one returns to one’s starting point from the other way. there is no boundary. We can’t envisage a hypersphere mentally, but we can .

The universe is *expanding* - the further away we look, the more rapidly the distant galaxies homogeneous means “the same at every point” and isotropic about a point p means that all directions at p are equivalent. The same recessiion of galaxies is seen wherever one is located in the universe. The expansion is uniform on a large scale,

$$R_{\alpha\beta\gamma\delta} = R_{\alpha\gamma} g_{\beta\delta} - R_{\beta\gamma} g_{\alpha\delta} + R_{\beta\delta} g_{\alpha\gamma} - R_{\alpha\delta} g_{\beta\delta} - \frac{1}{2} R (g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\delta} g_{\beta\gamma}). \tag{5.31}$$

Consider R^α_β which is a symmetric matrix and hence always has a complete set of orthogonal eigenvectors.

$$R^\alpha_\beta l^\beta = \lambda l^\alpha \tag{5.32}$$

Unless $R^\alpha_\beta = \delta^\alpha_\beta$ the eigenvectors would pick out a preffered direction. Hence, $R_{\alpha\beta} = K g_{\alpha\beta}$ for some K. Taking the trace we have

$$R_{\alpha\beta} = \frac{1}{3} R g_{\alpha\beta}, \tag{5.33}$$

substitutaing this into (M.-19) we obtain

$$R_{\mu\nu\gamma\delta} = \frac{1}{6} R (g_{\alpha\gamma} g_{\beta\delta} - g_{\alpha\delta} g_{\beta\gamma}). \tag{5.34}$$

The clocks move with space not through it.

Accelaration Equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \rho \left(1 + \frac{3w}{c^2} + \frac{\Lambda}{3} \right) \tag{5.35}$$

Distant objects can *appear* to move away faster than the speed of light. However, it is space itself which is expanding - the expansion of space is just like that of the balloon, and pulls the galaxies along with it. Special relativity refers to the relative speeds of objects passing each other, and can not be used to compare the relative speeds of distant objects.

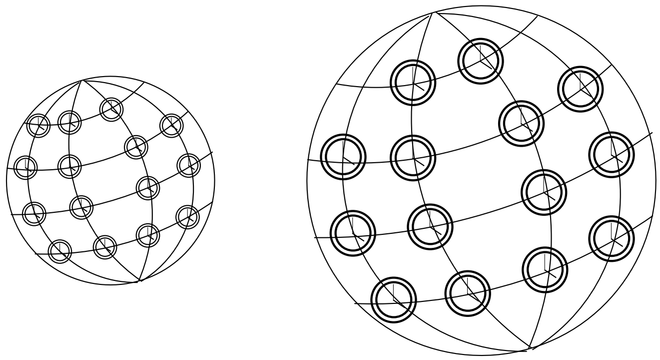

Figure 5.11: Cosmic time.

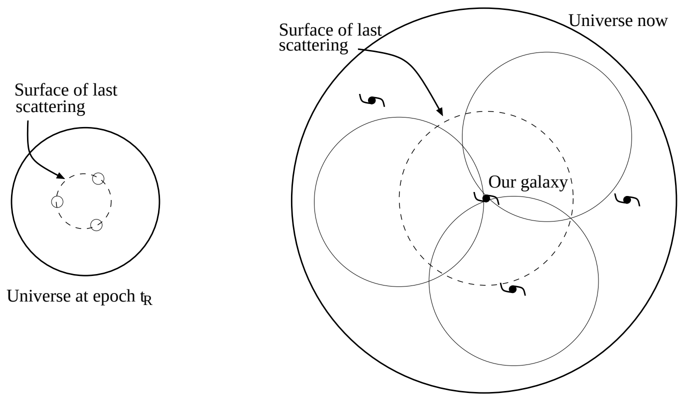

Figure 5.12: (a) Photon. (b) At the last scattering surface, the photons had a much higher frequency, which has been redshifted as the photons travel toward us.

The universe has expanded by a huge factor sinse the time of the Big bang, this initial fireball has dispersed by an absolutely enormous factor.

Standard Model of Cosmology

Models with a Cosmological Constant

5.10.2 Problems in Classical Cosmology

Dark Matter

There are two different basic ways of accessing the actual average density of matter in the universe.

There is missing matter that we can't see it referred to as dark matter.

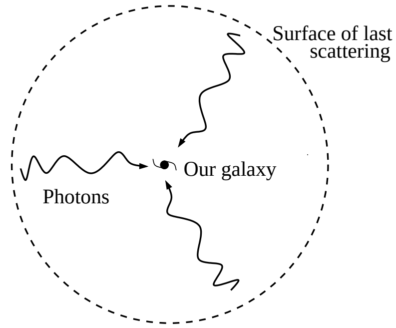

Figure 5.13: At the last scattering surface, the photons had a much higher frequency, which has been redshifted as the photons travel toward us.

Cosmological Constant

When positive it causes the universe to expand and to expand at an accelerating rate.

Ordinary matter causes the universe to contract because of mutual gravitational attraction of all the matter it contains.

Quantum theory appears to require a huge cosmological constant. If something is at rest it has definite position and momentum but this contradicts the uncertainty principle. As a consequence and degree of freedom cannot have zero energy. But a field

Quantum theory predicts a huge vacuum energy and by general relativity a huge cosmological constant. We know this is wrong.

However, we must keep in mind that the above considerations were made in the context of the semiclassical approximation of quantum fields on a fixed background. Let us go into more details. In this approximation the fixed background is given by the equation

$$G_{\mu\nu} + \Lambda^0 g_{\mu\nu} = 8\pi G \ < \Psi|\hat{T}_{\mu\nu}|\Psi > \tag{5.36}$$

where Λ^0 is a fundamental (or bare) cosmological constant and $|\Psi >$ is a "vacuum" state. Here the expectation of $\hat{T}_{\mu\nu}$ contributes to the observed cosmological constant and leads to a quantum state dependent effective cosmological constant Λ given by

$$\Lambda = \Lambda^0 - 2\pi G \ < \Psi|\hat{T}_{\mu\nu}|\Psi > \tag{5.37}$$

If the background is Minkowski or highly symmetric spacetime, the matter field vacuum is more or less unambiguous. In such cases

$$< \Psi|\hat{T}_{\mu\nu}|\Psi > = \rho g_{\mu\nu} \tag{5.38}$$

holds at least to leading order in $\hbar$, and can say that ρ in this equation contributes to the cosmological constant.

The observed value of the cosmological constant using the cold dark matter model is of the order 10^{-120} in Plank units, whereas the theoretical contribution to it from ρ alone, using a suitable cutoff, is far from this number. Indeed, to have a match between theory and experiment requires the finely tuned cacellation of Λ^0 and ρ to 120 decimal places. This is the usual statement of the cosmological constant problem.

Does the problem really exist at a more fundamental level in a non-perturbative context, outside the semiclassical approximation where there is no fixed background spacetime?

We should really be seeking a vacuum (or ground state) of a full non-perturbative- matter-geometry Hamiltonian derived from general relativity coupled to matter fields

Having determined a non-vanishing Hamiltonian, the task is then to find its ground state(s) $|q, \phi >_0$ and compute the ground state (or vacuum) energy. It is at this stage that there may be an emergent "vacuum energy problem" if the energy of the relevant state of $\hat{H}$ does not match the observed one.

From simply models it is demonstrated that at the non-perturbative level there is a relationship between the cosmological constant, time and the vacuum energy which is rather complex, and fundamentally different from what one would conclude from naive use of the semiclassical equation.

Dark Energy

The expansion of the universe appears to be accelerating, whereas, given the observed matter plus the calculated amount of dark matter, it should be decelerating. One proposed explaination is that there is this strange new energy, postulated to fit the data, thhe so called dark energy.

We do not know what the dark energy is. We only know about it because we can measure its effect on the expansion of the universe. It manifests itself as a source of gravitational attraction spread uniformaly through space. Its only effect it can have is on the average speed at which the galaxies move away from each other. In 1998 observations of supernovas in distant galaxies appeared to show that the expansion of the universe was accelerating in a way best explained by the existtence of dark energy.

The Flatness Problem

$$\Omega \approx 1 \text{ to an accuracey of } 10^2. \tag{5.39}$$

The Horizon Problem

The CMB looks more or less the same no matter which direction we look at in the sky. The CMB was emitted when the universe was about 3000,000 years old. Any area in the universe that is that has size gerater than 1 degree

Causal communication between two regions A and A' can eist only if they are within a distance $2R_H(t) = 4t$. Two regions separated by a proper distance greater than $2R_H(t)$ at epoch t could never have influenced each other. Hence, there is non a priori reason to expect points A and A' to have smilar physical environments.

5.10.3 Inflationary Universe

There is a big difference between this scenario and the old idea that the while universe was created at the same moment of time (Big Bang), in a nearly uniform state with indefinitely large temperature.

Inflationary cosmology has been regarded as an attractive scenario. Loop quantum cosmology is the first direct link to a propsed fundamental theory.

Power-law Inflation

[hep-ph/0505249]

$$a(t) = \text{const. } t^p \tag{5.40}$$

Slow-roll Approximation

$$H^2 = \frac{8\pi}{3M_p^2}V(\varphi) \tag{5.41}$$

5.10.4 Matter Driven Inflation

There are two equations that describe the evolution of a homogoneous scalar field in the model, the field equation

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi, \tag{5.42}$$

and Einstein's equation

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3M_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \tag{5.43}$$

The first equation is similar to the equation of motion of a damped harmonic oscillator, where instead of $x(t)$ we have $\phi(t)$. The thrid term is analogous to the term describing friction in the equation for a harmonic oscillator.

Inflation was a period of the early universe when $\dot{a}$ increses

$$\ddot{a} > 0 \tag{5.44}$$

The standard is a material with the unusual property of having negative pressure. However, such a particle plays a key role in particle physics theory of the electrweak force. for now we just The pressure and energy of the inflation field are given by

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) \tag{5.45}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) \tag{5.46}$$

where $V(\phi)$ is the potential energy. During the inflationary phase the inflation field . This is

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] \tag{5.47}$$

By substituting the scalar field pressure and energy into the fluid equation (**??**), we obtain the Klein-Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi) \tag{5.48}$$

where $'$ denotes differentiation with respect to ϕ. will sustain inflation while

$$\dot{\phi}^2 < V \tag{5.49}$$

i.e. while the kinetic energy is small. This is called the *slow-roll approximation*. Friedmann equation

$$H^2 \approx \frac{8\pi G}{3}V \tag{5.50}$$

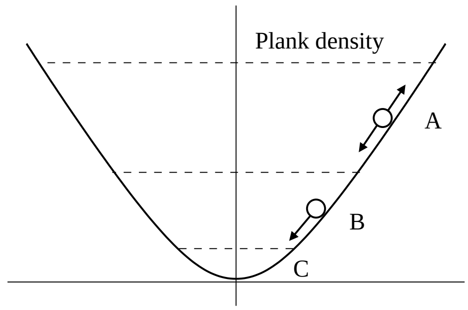

Figure 5.14: Motion of the scalar field in the theory with $\frac{m^2}{2}\phi^2$..

and the corresponding Klein-Gordon equation is

$$\dot{\phi}^2 \approx -\frac{V'}{3H}. \tag{5.51}$$

$|V'|$ is very small so H is nearly constant implying that $a(t)$ grows nearly exponentionally

$$\frac{\dot{a}}{a} \approx const. \tag{5.52}$$

The amount of inflation can be measured in terms of e-folding number N given by

$$N(t) = \ln\left(\frac{a(t_{after})}{a(t_{initial})}\right) \tag{5.53}$$

we need at least 50 e-folding to solve the hot Big Bang problems.

Reheating

During inflation the universe is supercooled by very rapid expansion. The radiation produced by this procces starts to *reheat* the universe, whereby the period of inflationary expansion gives way to the standard hot Big Bag phase.

The key point is that at the time of reheating, the entropy of the universe increases by a large factor, and this drives the universe towards spaciatial flatness, as can be seen from the FRW equations (M.-19).

5.10.5 Canonical Quantization

$$p_i = -i\frac{\partial}{\partial q^i} \tag{5.54}$$

Since the Hamiltoian is quadratic in p_i one obtains a second order differential equation called the Wheeler-DeWitt equation.

WKB solutions

the Schrödinger equation

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(x)\right]\psi(x) = E\psi(x) \tag{5.55}$$

with the WKB solution of the form

$$\psi(x) = e^{\frac{i}{\hbar}S(x)}, \tag{5.56}$$

may lead to the Hamilton-Jacobi equation in the limit $\hbar \to 0$,

$$\frac{1}{2m}(\nabla S)^2 + V(x) = E. \tag{5.57}$$

With the identification $p = \nabla S$ and $H = E$,

The same idea may be applied to quantum cosmology.

$$\Psi(\Omega, \phi) \sim \exp\left(\pm\frac{1}{3V(\phi)}(1 - e^{2\Omega}V(\phi))^{3/1}\right), \tag{5.58}$$

when the scale factor is large, the WKB solutions form

$$\Psi(\Omega, \phi) \sim \exp\left(\pm\frac{1}{3V(\phi)}(e^{2\Omega}V(\phi) - 1)^{3/1}\right). \tag{5.59}$$

5.10.6 Why does the Wheeler-DeWitt Equation fail?

$$-\frac{1}{6}l_P^4 a^{-1}\frac{d}{da}\left(a^{-1}\frac{d}{da}(a\psi(a, \phi))\right) = \kappa\hat{\mathcal{H}}_{matter}\psi(a, \phi) \tag{5.60}$$

quation still singular, possible rescue

boundary conditions,

e.g. $\psi(0) = 0$ motivaed by intuition (DeWitt)

no-boundary proposal, (Hartle-Hawking)

tunneling (Vileukia)

advantage: selects a unique state (up to norm) appropriate for a unique universe

but not enough to remove singularity:

a-spectrum continuous arbitrarity to $a = 0$ singulariy point of creation.

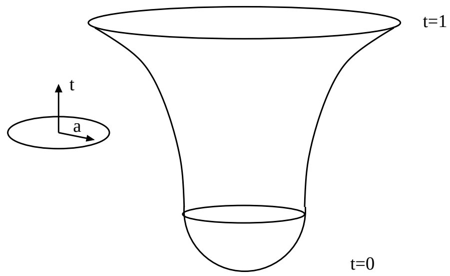

Figure 5.15: The universe emerging from nothing.

Wheeler-DeWitt Equation

Symmetry redudction at the classical level and then quantization. No direct relation to quantum gravity. Not derived from quantum gravity.

Loop quantum gravity is a well defined quantum theory at kinematic level (prior to imposing the Hamiltonian constraint) Symmetric reduction at this level after quantization.

5.11 Loop Quantum Cosmology

$$d\tau^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2). \tag{5.61}$$

that we have because we formulate the theory with triads instead of the metric. We can perform a Gauss gauge transformation without changing the metric they encode.

$$A_a \mapsto g^{-1} A_a g + g^{-1} \frac{\partial g}{\partial x^a}, \qquad E^a \mapsto g^{-1} E^a g \tag{5.62}$$

where A_a and E^a are the matrices $A_a^i \tau_i$ and $E^a \tau_i$ respectively.

We can rotate them with an $SO(3)$ element without changing the metric they encode.

$$
\begin{aligned}
qq^{ab} &= E_i^a E_j^b \delta_{ij} \\
&= \mathrm{tr}(E^a E^b) \\
&= \mathrm{tr}(g^{-1} E^a g g^{-1} E^b g)
\end{aligned} \tag{5.63}
$$

the Poisson brackets of any two functions f and g on this phase space is given by:

$$
\{f, g\} = \frac{\kappa\gamma}{3}\left(\frac{\partial f}{\partial c}\frac{\partial g}{\partial p} - \frac{\partial g}{\partial c}\frac{\partial f}{\partial p}\right) \tag{5.64}
$$

because of homogeneity and isotropy, we do not need all edges e and surfaces S. Symmetric connections A in $\mathcal{A}$ can be recovered knowing holonomies $h(e)$ along straight lines in $\mathcal{M}$. Similary,

Hamiltonian constraint uses volume operator, creation of loops - It is difficult to find solutions with interesting classical correspondence. There are quantum ambiguities. We can exploit simplified context of symmetric models.

The Hamiltonian constraint is a *difference equation*

$$
\hat{\mathcal{H}}|n >= f(n)(|n+4> -2|n> |n-4>) \tag{5.65}
$$

where $|n>$ iare eigenstates of the volume operator. Using the volume of the universe as "time", we take $\hat{\mathcal{H}}\psi = 0$ and interpret it as a discretized time evolution equation.

evolution equations don't breakdown even though the volume of the universe goes to zero!

$$
\mathcal{H} = -12\gamma^{-2}\kappa^{-1}(c(c-k) + (1+\gamma^2)k^2/4)\sqrt{|p|} \tag{5.66}
$$

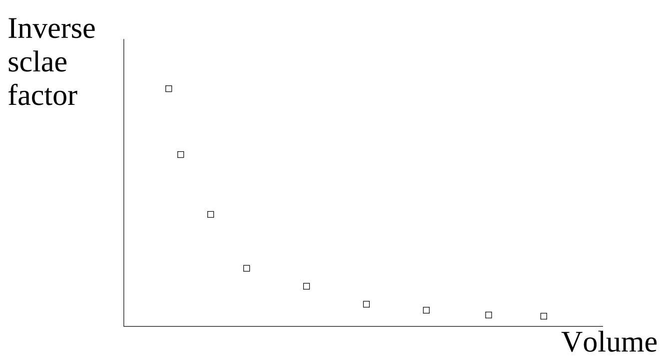

Figure 5.16: inversescale.

One has $|p| = a^2$ while $c = (k - \gamma\dot{a}$. If we insert this constraint equation $\mathcal{H} + \mathcal{H}_{\text{matter}}$

journal Advances in Theoretical and Mathematical Physics:

"The question of whether the universe had a beginning at a finite time is now 'transcended'. At first, the answer seems to be 'no' in the sense that the quantum evolution does not stop at the big-bang. However, since space-time geometry 'dissolves' near the big-bang, there is no longer a notion of time, or of 'before' or 'after' in the familiar sense. Therefore strictly, the question is no longer meaningful. The paradigm has changed and meaningful questions must now be phrased differently, without using notions tied to classical space-times."

• a^{-1} bounded (curvature cut-off)

• non-singular evolution (matter-independent, fixes ordering)

• dynamical intial conditions

• Wheeler-DeWitt quantum cosmology as continuum limit applies to realistic spacetimes

• Further insight from complicated models

• All except a finite number of degrees of freedom are "frozen". The shift function is zero and the lapse is homogeneous. N is still a function of τ, so that separation beween two succesive three-surfaces is still undetermined. Reparametrization invariance is what remains of general covariance of the full theory.

• Are there corresponding solutions in full theory?

• Only considering states that are symmetric at the microscopic level.

5.11.1 Details of the LQC Hilbert Space Representation

holonomies can be evaluated, starting from those associated to straight paths $\mu^0 e^i_a$, which gives

$$h(A)_a = \cos\frac{\mu c}{2} I + 2\sin\frac{\mu c}{2}\tau_a. \tag{5.67}$$

Hence, one may take almost periodic functions $N_\mu = e^{\frac{i\mu c}{2}}$ as a basis in the configuration space. The algebra generated by $\{N_\mu, p\}$ plays the same role as the holonomy-flux algebra. The analogous construction of full LQG turns out to be

$$\mathcal{H}_{kin} = L_2(\mathbb{R}_{Bohr}, d\mu_{Bohr}),$$

where $\mathbb{R}_{Bohr}$ is the so-called Bohr compactification of the real line. An orthonormal basis is given by the N_μ's, with measure

$$< N_{\mu'}|N_\mu > = \delta_{\mu'\mu}. \tag{5.68}$$

One has a countable set of basis vectors despite μ being conitinuous. The states are normalizable - this represents discreteness in a weaker sense.

The action of the operators is given by

$$N_\mu \psi(c) = e^{\frac{i\mu c}{2}} \psi(c), \qquad p\psi(c) = -i\frac{8\pi\gamma l_P^2}{3}\frac{d}{dc}\psi(c) \tag{5.69}$$

so that states $|\mu >$ can be defined so that

$$< c|\mu > = e^{\frac{i\mu c}{2}}, \qquad p|\mu >= \frac{8\pi\gamma l_P^2}{6}\mu|\mu > \tag{5.70}$$

We only consider the case $k = 0$.

The evaluation of the Hamiltonian constraint involves teh expression of the field strength F_{ij}^a, which can be obtained by the limiting procedure from holonomies.

The regualrization procedure involves the fixing of a minimum value $\mu = \overline{\mu}$, just as in the general case.

Since the Hamiltonian involves the volume operator V, it is advantageous to introduce the basis

$$|v > = \frac{2^{3/2}}{3^{7/4}} sign(\mu)|\mu|^{3/2}|\mu >,$$

such that

$$V|v > = \frac{2^{3/2}}{3^{7/4}}\left(\frac{8\pi\gamma}{6}\right)^{3/2} l_P^3|v||v > \tag{5.71}$$

Then the Hamiltonian H reads

$$H^{\overline{\mu}} = sign(p)\frac{4i}{8\pi\gamma^3\overline{\mu}^3 l_P^2}\sin^2\overline{\mu}c\left(\sin\frac{\overline{\mu}c}{2}V\cos\frac{\overline{\mu}c}{2} - \cos\frac{\overline{\mu}c}{2}V\sin\frac{\overline{\mu}c}{2}\right) \tag{5.72}$$

It is worth noting that the limit $\overline{\mu} \to 0$ does not exist. There are different ways of fixing $\overline{\mu}$; one choice is

$$\overline{\mu}|p| = 2\sqrt{3}\pi\gamma l_P^2,$$

such that the minimum area enclosed by a loop is given by the minimum area gap predicted by LQG.

The expression (5.72) can be made Hermitian by a certain factor ordering (though not necessary, improves solving the problem)

$$H^{\overline{\mu}} = sign(p)\frac{4i}{8\pi\gamma^3\overline{\mu}^3 l_P^2}\sin\overline{\mu}c\left(\sin\frac{\overline{\mu}c}{2}V\cos\frac{\overline{\mu}c}{2} - \cos\frac{\overline{\mu}c}{2}V\sin\frac{\overline{\mu}c}{2}\right)\sin\overline{\mu}c \tag{5.73}$$

which, when applied to $\psi(v)$, leads to the following difference equation

$$H^{\overline{\mu}}\psi(v) = f_+\psi(v+4) + f_0\psi(v) + f_-(v-4) \tag{5.74}$$

with

$$\begin{aligned} f_+(\mu) &= \frac{3^{5/4}}{2^{5/2}}\sqrt{\frac{8\pi}{6}}\frac{l_P}{\gamma^3/2}|v+2|\Big||v+1|-|v+3|\Big| \\ f_-(\mu) &= f_+(v-4), \qquad f_0(\mu) = -f_+(\mu) - f_-(\mu). \end{aligned} \tag{5.75}$$

Striking aspects of LQC is the replacement of the Wheeler-DeWitt differential equationn with a difference one. When a clock-like scalar field ϕ is introduced, the full Hamiltonian reads

$$H_{tot} = H + 8\pi G\frac{p_\phi^2}{|p|^{3/2}} \tag{5.76}$$

and canonical quantization leads to the follwoing difference equation for the dynamics

$$\frac{\partial^2}{\partial\phi^2}\Psi(v,\phi) = B^{-1}(v)H\Psi(v,\phi) = \Theta\Psi(v,\phi) \tag{5.77}$$

where the inverse volume operator's eigenvalues are

$$B(v) = \frac{1}{|p|^{3/2}} = \frac{3^{5/4}}{2^{3/2}}|v|\Big||v+1|^{1/3} - |v-1|^{1/3}\Big|^3. \tag{5.78}$$

The boundedness of the operator corresponding to $|p|^{3/2}$ was the first sign towards teh resolution of the cosmological singularity.

$$i\frac{\partial^2}{\partial\phi^2}\Psi(v,\phi) = -\sqrt{\Theta}\Psi(v,\phi) \tag{5.79}$$

The physical Hilbert space can be induced from $\mathcal{H}_{kin}$ by the group averaging technique.

If one starts from a semi-classical universe, described by a state sharply peaked around $\mu = \mu^* \gg \overline{\mu}$, and evolves it backwards in time, the state remains semi-classical during the evolution and a bounce occurs for $\mu \sim \overline{\mu}$, followed by an expansion phase.

5.11.2 On the Reliability of the Results of LQC

The scale operator, defined on the Kinematic Hilbert space, does not correspond to a Dirac observable as it does not commute with all the first class constraints generating the gauge transforamtions. Although the inverse scale factor operator commutes with the other constraints, it does not commute with the Hamiltonian constraint (except for very special choices of $\hat{C}_{matter}$). Thus the inverse scale factor operator is not a physical operator and its spectrum can not be considered as an indication of possible measurements of the spatial curvature. Consequently, the fact that this spectrum is bounded from above does not give a reliable indication that the kinematic singularity is avoided. In order for a legitiment conclusion to be drawn, a physical operator correspnding to an observable encoding spactial curvature is neccessary.

Although the inverse scale factor operator doesn't commute with the Hamiltonian constraint and does not qualify as a Dirac observable, its commutator with both the $SU(2)$ and the spatial diffeomorphism constraint vanishws. Thus, it constitutes an operator corresponding to a partially invariant partial observable [300]. As explained in chapter 1, one can turn such a partial observable into a Dirac obesrvable invariant under all the constraints. However, the spectrum of the operator corresponding to the Dirac observable will in general differ from the Kinematic spectrum, and sometimes drastically so. This means that the fact that the inverse scale factor operator is bounded from above cannot be used to produce reliable statements concerning the removal of the classical singularity in LQC [216].

5.11.3 Inflation from Loop Quantum Cosmology

effective Friedmann equation

A generalized Hamiltonian Constraint Operator in Loop Quantum Gravity and its simplest Euclidean Matrix Elements, Class.Quant.Grav. 18 (2001) 1593-1624, gr-qc/0011106

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{16\pi}{3} G a^{-3} \left(\frac{1}{2} a^{-3} p(3a^2/jl_P^2)^6 p_\phi^2 + a^3 V(\phi)\right). \tag{5.80}$$

Since the right hand side now depends on a for small a the classical behaviour, the dynamics is clearly modified.

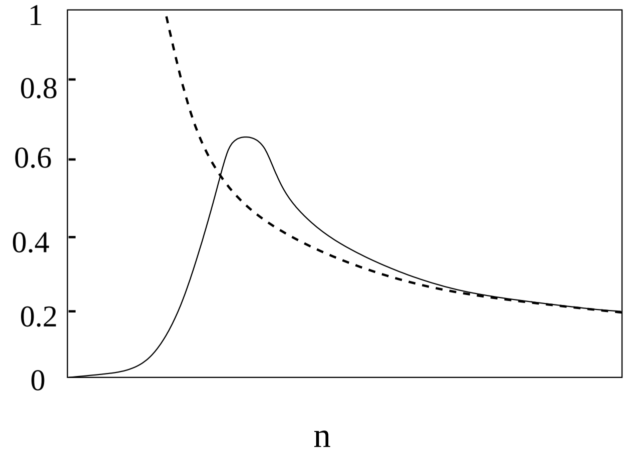

Figure 5.17: eigendensity Eigenvalues of the density operator for twp choices of the ambiguity parameter, compared to the classical expectation (thick, dashed).

The scalar field is quantum mechanical and will have characteristic quantum fluctuations. It is possible that these quantum fluctuatons to eventually manifest as classical density fluctuations.

5.12 Path Integral Formulation of Loop Quantum Cosmology

Recent developements have allowed for the construction of new spin foam models that are more closely related to the canonical theory. It is important to make connections to spin-foams by building path integrals from the canonical theory when possible.

5.12.1 Introduction

We construct a path integral for the exactly soluble Loop Quantum Cosmology starting with the canonical quantum theory.

The construction defines each component of the path integral. Each has non-trivial changes from the standard path integral.

We see the origin of singularity resolution in the path integral representation of LQC.

The structure of the path integral features similarities to spin foam models.

The path integral can give an argument for the surprising accuracy of the effective equations used in more complicated models.

5.12.2 LQC Canonical Theory as a Sum Over Histories

We examine the simply case of $k = 0$ FRW model with a scalar field

$$\nu = \epsilon \frac{a^3\ {}^oV}{2\pi \ell_p^2 \gamma}, \qquad b = -\epsilon \frac{4\pi\gamma G}{3\ {}^oV} \frac{p_a}{a^2} \tag{5.81}$$

With Poisson bracket

$$\{b, v\} = 2/\hbar$$

Classically they have range $(-\infty, \infty)$.

To simplify the constraint the lapse is chosen to be $N = a^3 N'$.

The phase space action is then

$$S = \int dt[\dot{b}v\frac{\hbar}{2} + p_\phi \dot{\phi} - \frac{N'}{2}(p_\phi^2 - 3\pi\hbar^2 G b^2 v^2)] \tag{5.82}$$

We will construct the path integral starting from the physical Hilbert space of Schrödinger LQC.

The physical states are solutions to the quantum Hamiltonian constraint.

$$\partial_\phi^2 \psi(\nu, \phi) = -\hat{\Theta}\psi(\nu, \phi) \tag{5.83}$$

The physical Hilbert space can be obtained by group averaging procedure.

Find that physical states satisfy a Schrodinger like equation.

$$-i\partial_\phi \psi(\nu, \phi) = \widehat{\sqrt{\Theta}}\psi(\nu, \phi) \tag{5.84}$$

The physical inner product is.

$$< \psi_1|\psi_2 > = \frac{\lambda}{\pi} \sum_{\nu=4n\lambda} \frac{1}{|\nu|} \overline{\psi}_1(\nu, \phi_0) \psi_2(\nu, \phi_0) \tag{5.85}$$

We have Schrodinger LQC written in the form of a Schrodinger equation with ϕ as time, so we apply the construction above to obtain a path integral.

Similar to the construction in non-relativistic quantum mechanics we want a path integral representation of the propagator.

$$< \nu' | e^{i\sqrt{\Theta}\Delta\phi} | \nu > = < \nu', \phi' | \nu, \phi > . \tag{5.86}$$

More generally we could construct the path integral from the definition of the physical inner product in terms of group averaging.

We have some knowledge of the exact propagator.

One can show that

$$< \nu', \phi' | \nu, \phi > = 0 \quad \text{if} \quad \nu' < 0 \quad \text{and} \quad \nu > 0 \tag{5.87}$$

This allows us to simplify the calculations by restricting to positive or negative ν.

The propagator can be written as an integral

$$< \nu', \phi' | \nu, \phi > = \frac{\lambda}{2\pi\nu} \int_0^{\pi/\lambda} db \, e^{\frac{b\nu}{2} - \frac{1}{\lambda} \tan^{-1}(e^{\Delta\phi} \tan(\lambda b/2))\nu'} + (\Delta\phi \rightarrow -\Delta\phi) \tag{5.88}$$

5.12.3 Vertex expansion: Reorganising the sum over histories

$$\begin{aligned} U_{\nu_M \nu_M} &= < \nu_M | e^{i\epsilon\mathcal{H}} | \nu_M > \\ &= 1 + i\epsilon < \nu_M | \mathcal{H} | \nu_M > + \mathcal{O}(\epsilon^2) \end{aligned} \tag{5.89}$$

$$\begin{aligned} U_{\nu_M \nu_{M-1}} &= < \nu_M | e^{i\epsilon\mathcal{H}} | \nu_{M-1} > \\ &= < \nu_M | \nu_{M-1} > + i\epsilon < \nu_M | \mathcal{H} | \nu_{M-1} > + \mathcal{O}(\epsilon^2) \\ &= i\epsilon \mathcal{H}_{\nu_M \nu_{M-1}} + \mathcal{O}(\epsilon^2) \end{aligned} \tag{5.90}$$

5.12.4 Derivation of Path Integral from the Canonical Theory

Step 1 - Split Exponetial/Insert Complete Basis

We split the exponential into N copies and insert a complete basis of ν between each.

$$1 = \frac{\pi}{\lambda} \sum_{\nu=4n\lambda} |\nu| \, |\nu >< \nu| \tag{5.91}$$

Giving

$$< \nu', \phi' | \nu, \phi > = \prod_{n=1}^{N-1} \left[\frac{\pi}{\lambda} \sum_{\nu_n} |\nu_n| \right] \prod_{n=1}^{N} \left[< \nu_n | e^{i\epsilon \widehat{\sqrt{\Theta}}} | \nu_{n-1} > \right] \tag{5.92}$$

Important difference: Instead of continuous integrals there are discrete sums over ν at each ϕ.

The next step is to compute each term of the product:

$$< \nu_n | e^{i\epsilon \widehat{\sqrt{\Theta}}} | \nu_{n-1} > \tag{5.93}$$

Step 2 - Evaluate Each Term

We want to evaluate each term to first order in ϵ

$$< \nu_n | e^{i\epsilon \widehat{\sqrt{\Theta}}} | \nu_{n-1} >$$

Problem: Even computing to first order in ϵ requires knowing the spectrum of Θ.

The resolution is to rewrite each term as

$$< \nu_n | e^{i\epsilon \widehat{\sqrt{\Theta}}} | \nu_{n-1} > = \int_{-\infty}^{\infty} dp_{\phi_n} |p_{\phi_n}| \Theta(p_{\phi_n}) \int_{-\infty}^{\infty} dN_n \frac{\epsilon}{2\pi\hbar} e^{ip_{\phi_n}\epsilon/\hbar} < \nu_n | e^{-i\epsilon \frac{N_n}{2\hbar}(p_{\phi_n}^2 - \hbar^2 \widehat{\Theta})} | \nu_{n-1} > \tag{5.94}$$

We then only need to evaluate the the following to first order in ϵ.

$$< \nu_n | e^{i\epsilon \frac{\hbar N_n}{2} \widehat{\Theta}} | \nu_{n-1} > \tag{5.95}$$

Evaluating this term is simple given the action of

$$< \nu_n | e^{i\epsilon \frac{\hbar N_n}{2} \widehat{\Theta}} | \nu_{n-1} > \;=\; \frac{\lambda}{\pi \nu_{n-1}} \Big(\delta_{\nu_n, \nu_{n-1}} - i\epsilon \frac{\hbar N_n}{2} \frac{3\pi G}{4\lambda^2} \nu_n \frac{\nu_n + \nu_{n-1}}{2} \times [\delta_{\nu_n, \nu_{n-1}+4\lambda} + \delta_{\nu_n, \nu_{n-1}-4\lambda} - 2\delta_{\nu_n, \nu_{n-1}}] + \mathcal{O}(\epsilon^2) \Big) \quad (5.96)$$

Expressed as an integral by writing the delta functions as integrals over b.

$$\frac{1}{\nu_{n-1}} \frac{\lambda^2}{\pi^2} \int_0^{\pi/\lambda} db_n e^{-i(\nu_n - \nu_{n-1})b_n/2} [1 + i\epsilon \frac{\hbar N_n}{2} \frac{3\pi G}{\lambda^2} \nu_n \frac{\nu_n + \nu_{n-1}}{2} \sin^2(\lambda b_n)] \quad (5.97)$$

Step 3 - Rexponentiate

Combining together the results from the previous slides and re-exponentiating the product we arrive at the path integral:

$$< \nu', \phi' | \nu, \phi > \;=\; \lim_{N \to \infty} \frac{1}{\nu_0} \prod_{n=1}^{N-1} \left[\sum_{\nu_n} \right] \prod_{n=1}^{N} \left[\frac{\lambda}{\pi} \int_0^{\pi/\lambda} db_n \int_{-\infty}^{\infty} dp_{\phi_n} |p_{\phi_n}| \Theta(p_{\phi_n}) \int_{-\infty}^{\infty} dN_n \right] \exp \frac{i}{\hbar} \sum_{n=1}^{N} \epsilon \Big[p_{\phi_n} - \frac{\hbar}{2} \frac{(\nu_n - \nu_{n-1})}{\epsilon} b_n - \frac{N_n}{2} (p_{\phi_n}^2 - \frac{3\pi G \hbar^2}{\lambda^2} \nu_{n-1} \frac{\nu_n + \nu_{n-1}}{2} \sin^2(\lambda b_n)) \Big] \quad (5.98)$$

Where the discretized action is

$$S_N = \sum_{n=1}^{N} \epsilon \left[p_{\phi_n} - \frac{\hbar}{2} \frac{(\nu_n - \nu_{n-1})}{\epsilon} b_n - \frac{N_n}{2} (p_{\phi_n}^2 - \frac{3\pi G \hbar^2}{\lambda^2} \nu_{n-1} \frac{\nu_n + \nu_{n-1}}{2} \sin^2(\lambda b_n)) \right] \quad (5.99)$$

There are non-trivial changes to the space of paths, measure, and action.

Allowed Paths in b, ν

Space of Paths: Defined by the range of integration at each time.

Paths $\nu(\phi)$ are discrete : $\nu(\phi)$ $(4\lambda, 8\lambda, 12\lambda, ...)$

Paths $b(\phi)$ are continuous, but bounded: $b(\phi) \in [0, \pi/\lambda]$

We are integrating only over discrete quantum geometries

Similar situation to that of spin foam models

Would try to define path integral over continuous fields ${}^4e$ and ${}^4A$

Instead integrate over discrete geometries.

The space of paths has been modified due to the kinematical structure of LQC.

The measure has changed - but is a natural measure on this space of paths.

Phase - Effective Action

The phase associated to each path is not the classical action.

$$S_N = \sum_{n=1}^{N} \epsilon \left[p_{\phi_n} - \frac{\hbar}{2} \frac{(\nu_n - \nu_{n-1})}{\epsilon} b_n - \frac{N_n}{2} (p_{\phi_n}^2 - \frac{3\pi G \hbar^2}{\lambda^2} \nu_{n-1} \frac{\nu_n + \nu_{n-1}}{2} \sin^2(\lambda b_n)) \right]$$

This is a discretized version of an effective action which contains non-perturbative quantum corrections

$$S = \int_{\phi}^{\phi'} d\phi \left[p_\phi - \frac{\hbar}{2} \dot{\nu} b - \frac{N}{2} \left(p_\phi^2 - \frac{3\pi G \hbar^2}{\lambda^2} \nu^2 \sin^2(\lambda b) \right) \right] \tag{5.100}$$

This is the effective action that well approximates the quantum dynamics.

Simplify Path Integral

Possible to integrate out variables to obtain a simpler expression?

Want configuration space path integral.

We can integrate out N and p_ϕ to obtain a path integral over b and ν only.

$$
\begin{aligned}
< \nu', \phi' | \nu, \phi > \ = \ & \lim_{N \to \infty} \frac{1}{\nu_0} \prod_{n=1}^{N-1} \left[\sum_{\nu_n} \right] \prod_{n=1}^{N} \left[\frac{\lambda}{\pi} \int_0^{\pi/\lambda} db_n \right] \\
& \exp \frac{i}{\hbar} \sum_{n=1}^{N} \epsilon \left[+ \sqrt{\frac{3\pi G \hbar^2}{\lambda^2} \nu_{n-1} \frac{\nu_n + \nu_{n-1}}{2}} \sin(\lambda b_n) \right. \\
& \left. \qquad - \frac{\hbar}{2} \frac{(\nu_n - \nu_{n-1})}{\epsilon} b_n \right]
\end{aligned}
\tag{5.101}
$$

Equivalent to solving the constraint → Path integral on constraint surface.

5.12.5 Configuration Path Integral's Connection to Spin Foams

The expansion in λ resembles the vertex expansion in group field theory.

5.12.6 Singularity Resolution

The path integral is then dominated by the extrema of the effective action

$$
S[v, b, p_\phi, N] = \int d\phi [p_\phi - \frac{\hbar}{2} b\dot{v} - \frac{N}{2}(p_\phi^2 - \frac{3\pi G\hbar^2}{\lambda^2} v^2 \sin^2(\lambda b))] \tag{5.102}
$$

The "classical solutions", x_{eff} to this action are the bouncing solutions.

The action can be computed along these bouncing solutions between v and v'.

$S_{eff}(x_{eff})$ is large in units of $\hbar \to 0$ loop corrections are negligible.

Provides an additional explanation for the accuracy of the effective equations.

5.13 Disordered Locality: A Possible Origin for Dark Energy and the Value of the Cosmological Constant

5.13.1 Introduction

the low energy limit may suffer from a disordered localilty characterised by identifications of far away points.

If macroscopic locality, as defined by the classical metric, is an approximation small departures could be indicators of an underlying quantum geometry. The consequences of spacetime being quantum mechanical can contribute to observable effects throughout the lifetime of the universe. Under some assumptions, it turns out that such phenomena at the macroscopis scale could lead to a contribution to the energy-momentum tensor that looks very much like a cosmological term, which has the potential of explaining the present value of the cosmological constant. However, there is currently no unique observational signiature that would distibguish this model from other dark energy pictures.

Any good model of quantumm gravity in which the classical metric is emergent (as with LQG) has to expalin why non-local connections are enough not to disrupt local physics. This has not yet been addressed, here it is just assumed that we have such a theory. Non-local connections on a lattice type structure can be both common cosmologically and very difficult to detect locally.

5.13.2 Notions and Hypothesis

Two kinds of locality:

Microlocality: connectivity of a single spin net graph causal structure of a single spin foam history.

Macrolocality: nearby in the classical metric that emerges

The issues: Semicassical states may involve superpositions of large numbers of graphs. In addition being semiclassical is a course grained, low energy property.

Could there not be mistmatches between micro and macrolocality?

It is not hard to see that they are generic.

For an emergent manifold scenario to work these mismatches must be suppressed.

Suppose they are, what scale measures how suppressed they are? What if it is the cosmological scale? Would there be observable consequences?

Hypothesis: the low energy limit of a theory with emergent manifold is characterised by a small worlds network

Suppose the ground state is contained by small proportion of non-local links (locality defects)??

If this room had a small proportion of non-local link, with no two nodes in the room connected, but instead connecting to nodes at cosmological distances, could we tell?

Studied the Ising model on a lattice contaminated by random non-local links.

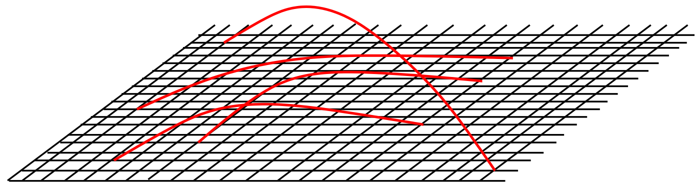

Figure 5.18: We begin with well-studied semiclassical states and contaminate them by adding a small proportion of links that are non-local in the classical metric q_{ab} without, however, weakening the correspondence between expectation values of coarse grained observables and the classical metric.

Conclusion was a small amount of disordered-locality would be hard to observe by local measurements.

What if the scale of disordered locality were cosmological?

Could there be observable consequences?

Hypothesis: disordered locality contributions to dark energy.

5.13.3 Disordered FRW Cosmology

The flat FRW metric

$$ds^2 = -\mathcal{N}^2 dt^2 + a^2(t) q^0_{ij} dx^i dx^j$$

We assum there is an underlying spin network $\Gamma(t)$,

That there are mostly local links

There are non-local links between N_{NL} pairs of nodes.

Macroscopically, N_{NL} pairs of points (x_i, y_i) identified. $i = 1, \ldots, N_{NL}$.

Both ends within present Hubble volume, distributed randomly.

N_{NL} evolves slowly

$$N_{NL}(a) = N_0 \left(\frac{a}{a_0} \right)^p$$

5.13.4 Energetics of Non-Local Interactions

We model interactions by a simple nearest neighbour coupling σ_i are degrees of freedom on each node (dimensionless).

$$H^{nodes} = -\epsilon \frac{1}{l_P} \sum_{<mn>} \sigma_m \cdot \sigma_n \qquad \epsilon = \pm 1.$$

This splits into a local and non-local part

$$H^{matter} = H^{local} + H^{NL}.$$

The local part becomes an ordinary matter coupling:

$$H^{local} = -\epsilon \frac{1}{l_P} \sum_{<mn>}^{local} \sigma_m \cdot \sigma_n \qquad \epsilon = \pm 1.$$

Effective scalar field at point x_n closest node n.

$$\phi(x_n) = l_P^{-1} \sigma_n \qquad \partial_a \phi(x_n) = \frac{1}{l_P^2}(\sigma_{n+\hat{a}} - \sigma_n).$$

This implies a local matter coupling

$$H^{local} = \frac{\epsilon}{2} \int d^3x \sqrt{q(x)} [q^{ab} \partial_a \phi \partial_b \phi - \mu^2 \phi^2]$$

$$\mu^2 = \frac{\sqrt{2}}{l_P^2}, \qquad \sum_n l_P^3 \to \int d^3x \sqrt{q}$$

The energetics of the non-local coupling:

Consider two regions $\mathcal{R}_1$ and $\mathcal{R}_2$ conected by N_{12} non-local links. The average field in a region is

$$< \sigma(x) >_{\mathcal{R}} = \frac{\int_{\mathcal{R}} \sqrt{q} \sigma}{\int_{\mathcal{R}} \sqrt{q}}.$$

We approximate the interaction energy between the two regions, averaged over possible ways to connect them with N_{12} non-local links

$$
\begin{aligned}
H^{\mathcal{R}_1\mathcal{R}_2} &= -\frac{\epsilon}{l_P}\sum_{I}^{x_I\in\mathcal{R}_1 y_I\in\mathcal{R}_2} \sigma(x_I)\cdot\sigma(y_I) \\
&\approx -\frac{\epsilon}{l_P}N_{12} \; <\sigma(x)>_{\mathcal{R}_1}<\sigma(x)>_{\mathcal{R}_2}
\end{aligned} \tag{5.103}
$$

This is called the annealing approximation in statistical mechanics.

Now consider the case that the two regions are the Hubble volume

$$
H^{NL} = -\frac{\epsilon}{l_P}N^{NL}(t) \; <\sigma>_a \cdot <\sigma>_a
$$

average field in the Hubble volume: $<\sigma>_a$.

Now put in the time dependence of $N_{NL}(t)$

$$
H^{NL} = -\frac{\epsilon}{l_P}\frac{N_0}{a_0^p}\left(\int_a d^3x\sqrt{q}\right)^{p/3} <\sigma>_a \cdot <\sigma>_a
$$

We choose $p=3$ to find

$$
H^{NL} = -\frac{\epsilon}{l_P}\frac{N_0}{a_0^3}\left(\int_a d^3x\sqrt{q}\right) <\sigma>_a \cdot <\sigma>_a
$$

This gives an effective action

$$
S^{NL} = \int dt\mathcal{N}H^{NL} = -\frac{\epsilon}{l_P}\frac{N_0}{a_0^3}\int_a d^4x\sqrt{-g} <\sigma>_a^2 .
$$

This in turn gives a contribution to the energy momentum tensor

$$
T^{ab}_{NL} = \frac{1}{\sqrt{-g}}\frac{\delta S^{NL}}{\delta g_{ab}} = -g^{ab}m^4 <\sigma>_a^2
$$

defines a mass

$$
m^4 = -\frac{\epsilon}{2l_P}\frac{N_0}{a_0^3}
$$

We see that $p = 3$ and $\epsilon = -1$ are necessary to get adrk energy $w = -1$. That is, the number of non-local links within the comoving volume increases in time proportionately to the comoving volume.

To match the dark energy we need (recalling $<\sigma>$ is dimensionless)

$$m^4 = \frac{N_0}{2l_P a_0^3} = \frac{10^{-120}}{l_P^4}.$$

Let us evaluate this at present. It gives:

$$N_{NL}(now) = 10^{-120}\left(\frac{a_{now}}{l_P}\right)^3 \approx 10^{60}.$$

A typical distance between ends of non-local links is 100km.

5.13.5 The Evolution of $N_{NL}(a)$ in a Simply Model of LQG

The Evolution of $N_{NL}(a)$, the number of non-local connections.

Let us consider a simple model of LQG with trivalent nodes and the related dual Pachner moves.

i) There are microscopic processes by which non-local links split into two and processes in which pairs annihilate.

ii) These must be in balance with the increase in volume which comes from 1 to 3 moves.

Exchange moves can increase the non-local edges.

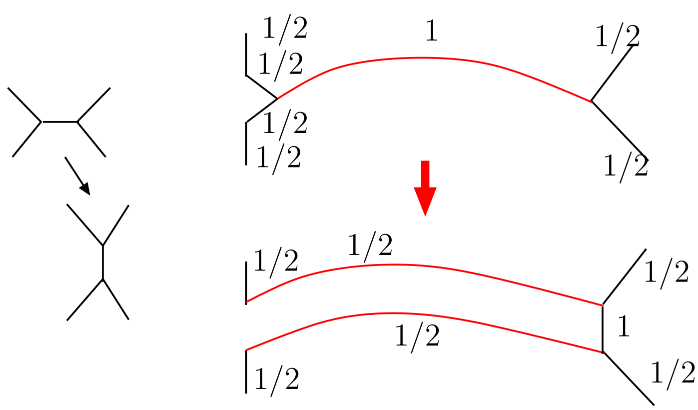

Figure 5.19: Exchange moves increasing the number of non-local edges.

The two left and two right edges can now evolve away from each other, leading to two non-local edges.

The probability for this to happen on each local move is

$$\alpha N^{NL}.$$

Exchange moves that decrease the non-local edges.

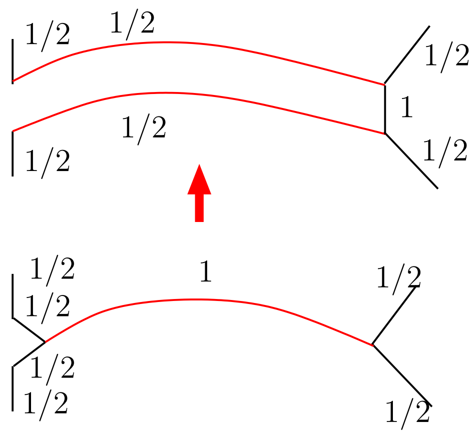

Figure 5.20: Exchange moves decrease the number of non-local edges.

This requires the inverse move on two non-local edges both of whose ends are coincident:

The probability is the probability that there are two non-local edges that coincide on each end times the probability that the move acts on one of them

$$-\beta(N^{NL})^3/V$$

The volume inceases by the net number of 1 to 3 moves over 3 to 1 moves.

$$\frac{dV}{dt} = VP_{1to3} = 3HV.$$

The change in the number of non-local links is dominated by

$$\frac{dN^{NL}}{dt} = P_{2to2}N^{NL}$$

Per node we have, on everage, $P_{2to2} = cP_{1to3}$ so:

$$\frac{dN^{NL}}{dt} = \frac{3H}{c} N^{NL}$$

$N^{NL} \sim$ volume implies that $c = 3$.

5.13.6 Conclusion

The conclusion is that disordered locality provides a possible explanation of dark energy.

5.14 Problems with Interpretations of Quantum Mechanics Applied to Quantum Cosmology

5.14.1 The Problem

Gravity governs the entire universe, so from a theory of quantum gravity we should formulate a quantum theory of cosmology.

Quantum gravity is to be applied to the universe as a whole. Issues are present.

$$\Psi_{univ} \tag{5.104}$$

There is nothing outside the universe. Do not have a satisfacotry physical interpretation of the state of the whole universe $|\Psi_{univ}>$ because it would only be accessible to an observer outside the universe.

We only understand the quantum mechanical evolution of ordinary systems.

5.14.2 Many Worlds Interpretation

The many worlds interpretation succeeds in letting the observer be part of the universe, but the world we observe is only a small part of reality.

5.14.3 Consistent Histories Interpretation

The probabiltiy for taking the branch I_n at time t_n *after* having gone through the branches $I_1, \ldots, I_{n-1}$ at $t_1, \ldots, t_{n-1}$ is given by

$$Tr\left(\hat{P}^n_{I_n}\hat{U}(t_n - t_{n-1})\hat{\rho}_{(I_1,t_1)\dots(I_{n-1},t_{n-1})}\hat{U}(t_n - t_{n-1})^{-1}\hat{P}^n_{I_n}\right) \tag{5.105}$$

and when this branch is chosen, the density matrix for that branch is given by

$$\hat{\rho}_{(I_1,t_1)\dots(I_n,t_n)} = \frac{\hat{P}^n_{I_n}\hat{U}(t_n - t_{n-1})\hat{\rho}_{(I_1,t_1)\dots(I_{n-1},t_{n-1})}\hat{U}(t_n - t_{n-1})^{-1}\hat{P}^n_{I_n}}{Tr\left(\hat{P}^n_{I_n}\hat{U}(t_n - t_{n-1})\hat{\rho}_{(I_1,t_1)\dots(I_{n-1},t_{n-1})}\hat{U}(t_n - t_{n-1})^{-1}\hat{P}^n_{I_n}\right)} \tag{5.106}$$

5.14.4 Problems with the Consistent Histories Interpretation

The set of questionswe can ask are consrtained by having to be solutions of certain equations. Although we can solve the equations that determine the quantum states of the universe, it is much more difficult to determine the questions that can be asked of the theory and it is unlikely that this can ever be done.

5.15 Relational Quantum Cosmology

There are many quantum theories, corresponding to as many observers there are. They are interelated, because when two observers can ask the same question they should get the same answer.

Crane

Fontini

Smolin

Isham and Butterfield

Classical logic demands that every statement be either true or false. Topos is a category that has properties in common with the category of sets **Set**.

5.15.1 Relational Quantum Cosmology a la Smolin

[186]

[185] (Crane)

5.15.2 Rovelli's Relation Quantum Mechanics

quickly introduce rovelli's **relation quantum mechanics**.

In conventional quatum mechanics a measurement entails: the superpostion principle holds alternative, and eventually mutually exclusive possibilities, right until a measurement is made that suddenly selects one of them alone as the realized actuality on this occasion.

There are many different mathematical descriptions, each corresponding to what each different observer can see. Each is incomplete, because no observer can see the whole universe. Each observer

when they ask the same question, they must agree.

Take quantum mechanics to be a fundamental description, that is, applies equaly to both the microscopic and the macroscopic world. Also that there is no special observer which can collapse a wavefunction. The conceptual difficulties boils down to one issue, an apparent discord: a system has a determined outcome according to one observer, while at the very same time is still in a quantum superpostion according to a second observer, that is, until the second observer makes a measurement. Now, what exactly is our reason for regecting this as a description of reality? There is no experimental evidence that telling us that such a discord is impossible, infact the very formulation of quantum mechanics rules out the possibility for any such experiment to be devised! Quantum mechanics is internally consistent because the time at which the wavefunction collapsed does not affect the predictions of the final observations. What is stopping us from accepting this as a description of reality isn't any logical insonsistenceis in the fromulation of quantum mechanics or any empirical evidence at the microscopic or macroscopic level, it is a philosophical pregudece!

"... the common unease with taking quantum mechanics as a fundemental description of nature could derive from an incorrect notion - as the unease with the Lorentz transformations before Einstein derived from the notion of observer independent time."

"... We suggest that the incorrect notion that generates the unease with quantum mechanics is the notion of observer-independent state os a system.

different obsevers may give different descriptions of the same sequence of events.

Smolin incorporates R to address problem of quantum cosmology

The most complete information one can have of the universe is the collection of these incomplete, but mutually consistent quantum state descriptions. There is no way in principle of formimg one wavefunction for the whole universe.

advantages when it comes to metaphysical issues many worlds

Instead of ***many worlds*** there is one universe with ***many observers***.

5.15.3 Causal sets

The internal description of a causal set: What the universe looks like from the inside [366]

Quantum causal histories [367]

An insider's guide to quantum causal histories [368]

Planck-scale models of the Universe [369]

Evolution in Quantum Causal Histories [370]

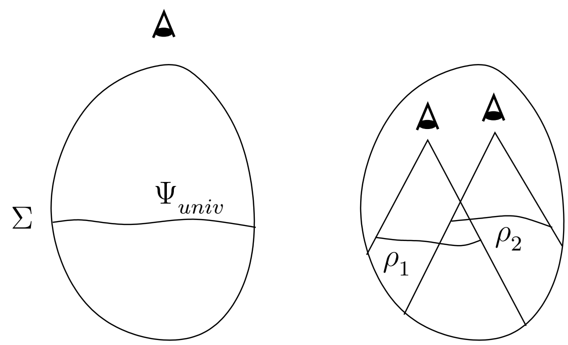

Figure 5.21: cosmologyeyes.

5.15.4 Discrete Quantum Gravity Applied to Cosmology

A very simple example

$$L = E\dot{A} + \pi\dot{\phi} - NE^2(-A^2 + (\Lambda + m^2\phi^2)|E|) \tag{5.107}$$

The system has four hase space variables and one constraint. Therefore it has two independent observables ($\{\mathcal{O}, C\}$):

$$\mathcal{O}_1 = \phi, \quad \mathcal{O}_2 = \pi + \frac{2}{3}\frac{m^2\phi}{\Lambda + m^2\phi^2}AE. \tag{5.108}$$

$$L(n, n+1) = E_n(A_{n+1} - A_n) + \pi_n(\phi_{n+1} - \phi_n) - N_n E_n^2(-A_n^2 + (\Lambda + m^2\phi_n^2)|E_n|) \tag{5.109}$$

Notice that the discrete theory has four phase space degrees of freedom instead of the two of the continuum theory. The additional degrees of freedom charactrize the step of the discretization and encode remnants of the gauge invariance in the discrete theory.

Although the graphs suggest that the triad goes to zero at $n = 0$ and therefore one has a singularity this is not the case.

We here show the approach to the singularity in the discrete and continuum case. The discrete theory has a small but non-vanishing triad a $n = 0$. R. Gambini and J. Pullin gr-qc/0212033

The rate of contraction/expansion changes when going through the big crunch/bang. Question: is that a remnent of the reparametrization invariance or does it have physical consequences?

The answer to this queation is related with the existence of more degrees of freedom and therefore more constants of motion. In fact, the discrete canonical transformation is singuluar for $A = 0$. If one tries to introduce a generator of this evolution:

$$A_{n+1} = A_n + \{A_n, \mathcal{H}_n\} + \frac{1}{2!}\{\{A_n, \mathcal{H}_n\}, \mathcal{H}_n\} + \cdots \tag{5.110}$$

$$\mathcal{H}_n = \frac{\mathcal{C}_n^2}{4\Theta A_n}\left[1 + \sum_{k=1}^{\infty} a_k \left(\frac{\mathcal{C}_n}{A_n^2}\right)^k\right] \tag{5.111}$$

H_n diverges for $\left(\frac{\mathcal{C}_n}{A_n^2}\right)^k > 2$. This happens for $n = 0$ when the system goes through the singularity.

H_n, which is constant on each region characterizes the spacing of the discretization in an invariant way, and in that sense suggest a procedure for taking the continuum limit.

Tunneling through the singularity the lapse gets modified and therefore the “lattice spacing” before and after is different. in lattice gauge theories the sacing is related to the “dressed” values of the fundamental constants a mechanism for fundamental constants to change when tunneling through a singularity.

Have some bare charge then you have an effetive renormalization but the effective renormalization of the lattice.

It may provide a mechanism for changing the values of the fundamental constants: They argue that this result for the particular cosmological model, this feature of tunneling through a singularity should to exists in a variety of models. This can be extrapolated to the internal of black holes. Each black hole will have its singularity replaced by tunneling into a new universe, in which the dressed value of the fundamental constants will be different. This allows to construct a picture of the universe in which “evolution” takes place every time a black hole is formed, as was the original proposal of Smolin in “The life of the cosmos”.

5.16 Summary

• Black hole entropy: very detailed calculation. Microscopic degrees of freedom for black-hole entropy. Once this parameter is fixed the correct formula for any no extremum blackhole (except rotating ones maybe)

• Removle of cosmological singurlarity. Evolutional equations don't break down at the place where the classical singularity is.

• Initial conditions are derived rather than guessed at - the evolution equations supplies the boundary conditions. Perhaps not suprising as the constraint equations are admissible conditions on the initial data.

• First direct derivation of inflation from a candidate for quantum gravity. Due to a quatnum geometry effect in the early kinematic dominated universe.

• Some features in the minisuperspace are shared with the full theory. Allows proper investigation of the dynamics of minisuperspace that could shed some light on the dynamics of the full theory.

• Disordered locality as a possible contribution to dark energy with the possible consequence of a naturally small vacuum energy.

• Relational Quantum Cosmology