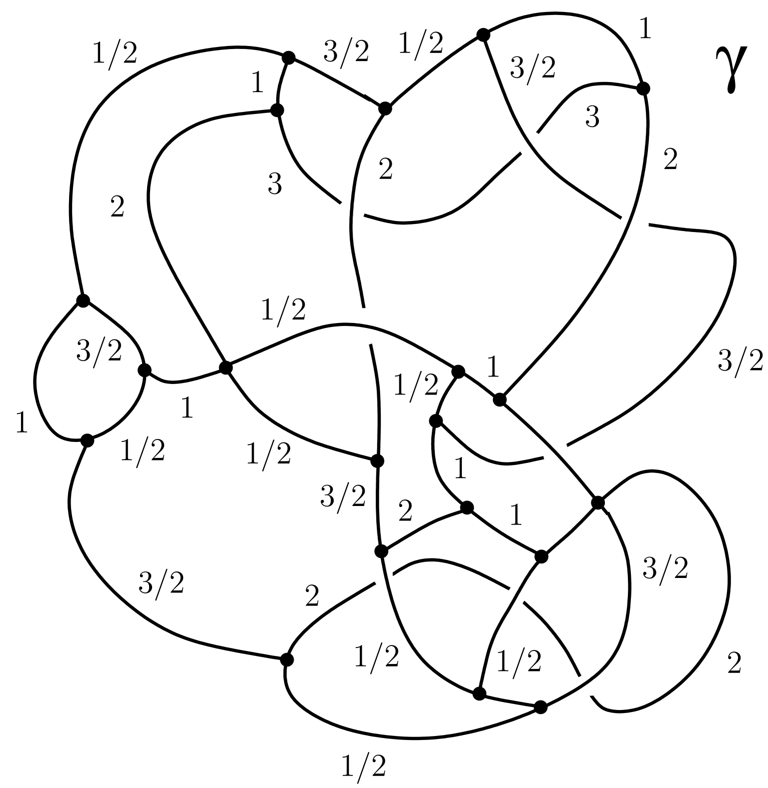
Volume II
Loop Quantum Gravity



Draft version

By

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Terminology and Notation

Here is a list of symbols.

[,]	commutator
{ , }	Poisson bracket
†	Hermitian conjugation
:=	definition
$\equiv$	identity
$\stackrel{*}{=}$	only true in a special coordinate system
iff	If and only if
η_{ab}	Minkowski metric
$\eta(x)$	test function of a variation of action
$\mathcal{A}$	space of gauge fields or area
$A_\mu(x)$	Yang-Mills connection
D_μ	covariant derivative
$\mathcal{M}$	spacetime manifold
$\mathbf{M}$	The Master consraint
$\hat{\mathbf{M}}$	The Master consraint operator
$\omega^\alpha_{\mu\beta}$	spin connection
$\mathcal{C}$	constraint surface in phase space
S	labells spin-network
s	equivalent class of spin-networks under the action of Diff denoted $s-$ knots
$s(S)$	denotes equivalent class S to which belongs
g_{ab}	spacetime metric
K_{ab}	extrinsic curvature of Σ
G_{ab}	Einstein tensor
T_{ab}	The energy-momentum tensor
e^a_I, E^a_i	tetrad and triad
$\mathcal{L}_t$	Lie derivative with respect to t
n_a	unit normal to Σ_t
N, $(\tilde{N})$	lapse function (density)
N^a	shift vector on Σ
$\Omega_{\alpha\beta}$	sympletic form
$\mathcal{A}/\mathcal{G}$	space of gauge fields moduli gauge transformations
$[A]$	gauge equivalence classe of the connection A
$\mathcal{HA}$	the holonomy algebra
$\overline{\mathcal{HA}}$	the completion of the holonomy algebra in the norm $\|f\| := \sup_{[A]\in\mathcal{A}/\mathcal{G}} \mid f([A]) \mid$
$\overline{\mathcal{A}/\mathcal{G}}$	spectrum of $\overline{\mathcal{HA}}$

Preface

> So how do you go about teaching them something new? By mixing what they know with what they don't know. Then, when they see in their fog something they recognize they think, "Ah, I know that!" And then it's just one more step to "Ah, I know the whole thing." And their mind thrusts forward into the unknown and they begin to recognize what they didn't know before and they increase their powers of understanding"
>
> PICASSO

Warning: We are sure there are lots of mistakes in these notes. Use at your own risk! Corrections and other feedback would be very appreciated.

These notes are meant to be supplementary to Loop quantum gravity literarture. It is explicit (fills in all the gaps normally left out of the calculations);It is a self-contained and bringing together of relavant maths. Blow-by-blow account. Present material in lauguage more familiar to a broarder community of theoretical physists. To which modern coordinate-free differential geometry is unfamiliar and possibly daunting.

Style of the Book:

The report is mainly aimed at theoretic physicists who are non-experts. fill in calculations an details that are skipped over in th litrature. people who we have tried to make it accessible to people who are not familiar with modern methods and will have no need to apply these methods in their field of study. for example coordinate-free diffential gometry

I should stress at the beginning that I am a physicist and not a mathematician will not always be at a level of rigour that would satisfy a proper mathematical physicist.

This report assumes the reader is somewhat familiar with special relativity and it would help if he/she has knowledge with general realtivity at the undergraduate level. The treatment is self-contained in that an a priori knowledge is not assumed. Try to bring together many useful results. Many proofs are added and others expanded on to make them more accessible.

The many excellent expositions of Loop-quantum gravity

The initial idea was to try to present the subject at a level lower than in most of the litrature. It may take the reader rather long time to make his/her way through the report.

the report offer something of value for the mathematically inclined and the not so alike.

[32] T. Thiemann, Introduction to Modern Canonical Quantum General Relativity, [gr-qc/0110034]

I'm not really an expert and I will try to indicate where my understanding is a bit more iffy. At the moment it is an enthusiastic if not entirely reliable account, which I hope to improve on this through critisms, suggestions...

The purpose of these notes, not to replace any book or paper. You should turn to other source for other explainations.

Nevertheless, these notes written to be reasonably self-contained and comprehensible.

I make no claim to originality in these notes.

Often follows quite closely the treatments of

I have used a number of sources.

though it is not indispensable and understanding for the report.

This discussion is intended to provide only a passing familiarity with the scheme to allow the unfamiliar reader to follow certain calculations and to have a general understanding of the results.

Acknowledgments

Dedicated

Introduction

the beginning of the revolutionary contributions to physics by Einstein,

An attempt to apply the principles of quantum mechanics to general relativity.

The problem of merging gravitation and quantum mechanics is extrememly difficult - it has defied solution for over seventy years. Classical theory developed by Einstein took 10 years. the principles of quantum mrchanics and general relativity.

Canonical quantization is the oldest non-perturbative approach to quantization of general relativity.

"Is it just me or is quantum gravity easier nowadays?"

Didn't think he would see the completion of quantum gravity in his own life time. dead and buried. There is now hope (not a word usually assocciated with quantum gravity for a lot of people).

In spite of these compelling features... Quantizing gravity equivalent to finding a general solution to the classical field equations. The techniqual difficulties seemed even worse than non-renormalization of perturbative quantum gravity. Seem hopelessly difficult.

A school of thought has a long history in studies of quantum gravity. The viewpoint here is that it may well be possible to quantise pure general relativity consistently, and in a way that respects the geometrical framework of the classical theory, but to do so requires the use of techniques that are quite different from the weak-field perturbative methods that, for example, have dominated most particle-physics based approaches to quantum gravity. Much effort has been devoted to finding such nonperturbative schemes, and in this paper we will be concerned with a particular one that has evolved from the introduction of a new set of canonical variables to describe the phase space of classical general relativity.

A matter of getting the right perspective on the problem. General relativists have argued for decades that gravity must be quantized non-perturbativly - easily said that done!!

List of acheivements:

(1) Predcition of quantized strectrum for area and volume operators.

(2) Black hole entropy from first principles for physical black holes in that

(i) non-exremal black holes non-rotating black holes with Maxwell charges and dilatonic charges (1998), and recently has even been extended to rotating black holes and black holes with arbitrary disortions, also

(ii) definition of black hole applies restrictions to horizon geometry only and not to the bulk spacetime outside the black hole. As a consequence, we can have time varying dynamics outside black hole; also means results extend to cosmological horizons too.

(3) Resolution of the big bang singularity.

(4) initial boundary conditions for the universe are determined, rather than guessed at.

(5) New mechansim for inflation from quantum geometry.

(6) A mathematically consistent quantum theory of gravity coupled to the standard model - explicitly demonstrated to be finite (1997). However serious worries about whether it is physically consistent - in particular whether it has the correct classical limit.

Progress in conceptual no less dramatic

(1) Relational quantum mechanincs.

If it ultimently the case that it does not lead to the correct theory, we will have leared many things of value along the way.

The book is primary on loop quantum gravity, however: Brian Greene in The Fabric of the Cosmos he says

"If I were to hazard a guess on future developments, I'd imagine that the background-indpendent techniques developed by the loop quantum gravity community will be adapted to string theory, paving the way for a string formulation that is background indepndent. And that spark, I suspect, that will ignite a third susperstring revolution in which, I'm optimistic, many of the remaining deep mysteries will be solved".

Paths through the report

Introductory book on general relativity [?]

T. Thiemann, Introduction to Modern Canonical Quantum General Relativity, [gr-qc/0110034] [32]